LESSONS IN ELECTRICITY AND MAGNETISM

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LESSONS IN
ELECTRICITY AND MAGNETISM

A TEXT BOOK FOR COLLEGES
AND TECHNICAL SCHOOLS

BY

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PREFACE.

This book and a companion volume entitled *Lessons in Mechanics* have been arranged to meet the needs of the two-year schedule in elementary physics which has been recently adopted in some of our technical schools, and, in our opinion, most of the added emphasis which this two-year schedule involves should be directed towards the mathematical side, or, let us say, the mathematical bottom of the subject.

The two-year schedule in elementary physics, beginning with the freshman year, means that teachers of physics cannot base their work upon college courses in mathematics to the extent that might have been possible heretofore; but we believe, nevertheless, that teachers of physics can and should use the more powerful mathematical methods from the beginning. Accordingly we have used differential and integral calculus throughout these new texts.

The central idea underlying these new texts is that mathematical training must be accomplished by the combined efforts of teachers of mathematics and teachers of physics, and it is hoped that these texts may help to bring our teachers of physics to what we believe to be their proper function in the important and difficult matter of mathematical training. If our students were to come from their mathematics teachers fully able to use their mathematics there would be nothing left for the rest of us to do!

The central purpose of these new texts is to facilitate classroom work. Descriptive and explanatory material has been reduced to a minimum, because students will not read more than is absolutely necessary; the development of every topic leads as directly as possible to illustrative numerical problems; and everything is arranged in strict lesson order.

The authors are indebted to members of the staff of the Department of Physics of the Massachusetts Institute of Technology for many suggestions that have been helpful in the preparation of this book.

THE AUTHORS.

IMPORTANT BOOKS AND HELPFUL REFERENCES.

Important Books for the Beginner.

Hadley's *Electricity and Magnetism*, London, Charles Griffen & Co., 1904.

J. J. Thomson's *Elements of Electricity and Magnetism*, Cambridge University Press, 1909.

Poynting and Thomson's *Electricity and Magnetism*, London, Charles Griffen & Co, 1914

Franklin and MacNutt's *Advanced Electricity and Magnetism*, Bethlehem, Pa. Franklin and Charles, 1915.

Lodge's *Modern Views of Electricity*, London, Macmillan & Co., 1907.

Helpful References for the Beginner.

Unusually good discussions of the following topics are given in Poynting and Thomson's *Electricity and Magnetism.*

Properties of dielectrics and measurement of inductivity, pages 120–133.

Pyro-electricity and piezo-electricity, pages 148–163.

Para-magnetism and dia-magnetism, pages 203–207 and 282–300.

Magnetism and light (a group of interesting effects), pages 320–340.

An instructive discussion of the mechanical conceptions of electromagnetic action is given on pages 245–263 of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

A good discussion of thermoelectric currents is given in J. J. Thomson's *Elements of Electricity and Magnetism*, pages 506–518.

See also Nichols and Franklin's *Elements of Physics*, Vol. 2, pages 216–221; New York, The Macmillan Co., 1898.

A very good discussion of electrolysis is given on pages 176–406 of W. C. D. Whetham's *Theory of Solution*, Cambridge University Press, 1902.

Important references for the student of electrical engineering

An exhaustive discussion of the magnetic properties of iron is given in J. A. Ewing's *Magnetic Induction in Iron and other Metals*, The Electrician Publishing Co., London, 1898; and a very good discussion of same by Dr. Saul Dushman in Vol. 19 of the *General Electric Review*. A good elementary discussion of the molecular theory of magnetism is given in Poynting and Thomson's *Electricity and Magnetism*, pages 192–202.

A full discussion of Fortesque's theory of insulator design is given on pages 186–191 of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

The theory of graded cable insulation and a discussion of the influence of heterogeneity of dielectric on dielectric stresses is given on pages 156–158 of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

Inductance of a transmission line is discussed on pages 60–62 of Franklin and MacNutt's *Advanced Electricity and Magnetism*

Capacity of coäxial cylinders and parallel cylinders (cables and transmission lines) is discussed on pages 168–185 of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

A simple introduction to the mathematical theory of electric waves, including a full discussion of transmission line surges and a discussion of wave distortion and line loading is given in Chapter IX of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

A good discussion of Ship's Magnetism and the Compensation of the Ship's Compass is to be found on pages 85–100 of Andrew Gray's *Treatise on Magnetism and Electricity*, Vol. I, London, Macmillan & Co., 1898; and a simpler discussion is given on pages 104–120 of Franklin and MacNutt's *Advanced Electricity and Magnetism.*

Books for Additional Study.

A favorite treatise on Electromagnetic Theory among advanced students is Abraham & Föppl's *Theorie der Electrizität*, 2 vols. Leipzig, B. G. Teubner, 1904.

The classical treatises on Electromagnetic theory are: (*a*) Faraday's *Experimental Researches*; (*b*) Maxwell's *Electricity and Magnetism*, 2 vols., 3d edition, Oxford, 1904; (*c*) Hertz's *Electric Waves*, translated by D. E. Jones, London, Macmillan & Co., 1893, and (*d*) Heaviside's *Electromagnetic Theory*, 2 vols., London, The Electrician Pub. Co., 1890.

Helps in the study of general theory

A very helpful discussion of divergence and curl and of scalar and vector potential is given in chapter IX of Franklin, MacNutt and Charles' *Calculus*, Bethlehem, Pa., Franklin and Charles, 1909.

A good introduction to the study of electric waves is to be found in chapter IX of Franklin and MacNutt's *Advanced Electricity and Magnetism;* see also Franklin's *Electric Waves*, Bethlehem, Pa., Franklin and Charles, 1909.

Every student should know of the following most important collections of physical and chemical data.

1. *Tables Annuelles Internationales de Constants.*
2. *Physikalisch-chemische Tabellen*, Landolt-Börnstein.
3. *Smithsonian Tables* (Fowle); publication No. 2269.

THE MORE IMPORTANT BOOKS ON ELECTRONS AND THE ELECTRON THEORY AND ON RADIOACTIVITY.

The electron theory is at present the most fruitful branch of theoretical physics, and the following are the most helpful books on this subject.

Conduction of Electricity through Gases, J. J. Thomson, Cambridge University Press, 1903 and 1906.

Electricity and Matter, J. J. Thomson, New York, Charles Scribner's Sons, 1904.

Rays of Positive Electricity, J. J. Thomson, London, Longmans, Green & Co., 1913.

Electrons, Oliver Lodge, London, Geo. Bell & Sons, 1909.

The Electron, R. A. Millikan, University of Chicago Press, 1917.

Theory of Electrons, H. A. Lorentz, Leipsig, B. G. Teubner, 1909.

The Emission of Electricity from Hot Bodies, O. W. Richardson, London, Longmans, Green & Co., 1916.

Photo-Electricity, H. Stanley Allen, London, Longmans, Green & Co., 1913.

Relativity and the Electron Theory, E. Cunningham, London, Longmans, Green & Co., 1915.

Radioactivity, Frederick Soddy, London, The Electrician Publishing Co., 1904.

The Interpretation of Radium, Frederick Soddy, New York, G. P. Putnam's Sons, 1912.

The Chemistry of the Radio Elements, Frederick Soddy, London, Longmans, Green & Co., 1914.

Radioactive Substances and their Emanations, Ernest Rutherford, Cambridge University Press, 1913.

Practical Measurements in Radioactivity, W. Makower and H. Geiger, London, Longmans Green & Co., 1912.

NATIONAL ORGANIZATIONS AND SOCIETIES RELATING TO PHYSICAL SCIENCE AND ENGINEERING.

The American Physical Society.

Every advanced student of physics and everyone who is interested in physical research of any kind should know about the *American Physical Society* and its official organ of publication, *The Physical Review*. The Secretary of the Society is Professor D. C. Miller of the Case School of Applied Science, Cleveland, Ohio; and the managing editor of the *Review* is Professor Frederick Bedell of Cornell University, Ithaca, N. Y.

The United States Bureau of Standards.

Every student of physics and chemistry and every student of engineering should know about the Bureau of Standards. A letter addressed to the Director, Dr. S. W. Stratton, Washington, D. C., will bring full information concerning the activities of the Bureau and a list of its publications. The scientific and technological papers of the Bureau are published chiefly in the *Bulletin of the Bureau of Standards*, and the *Bureau Circulars* are occasional publications.

ENGINEERING SOCIETIES.

Name.	Field.	Headquarters Office.
1. American Society of Civil Engineers.	Civil Engineering.	29 W. 39th St., New York City.
2. American Society of Mechanical Engineers.	Mechanical Engineering.	29 W. 39th St., New York City.
3. American Institute of Electrical Engineers.	Electrical Engineering.	29 W. 39th St., New York City.
4. American Institute of Mining Engineers.	Mining Engineering and Metallurgy.	29 W. 39th St., New York City.
5. American Chemical Society.	Chemistry and Chemical Industry.	Dr. Chas. L. Parsons, Sec'y, Box 505, Washington, D. C.
6. Illuminating Engineering Society.	Illumination.	29 W. 39th St., New York City.
7. Institute of Radio Engineers.	Radio telegraphy and telephony.	Mr. Davis Sarnoff, Sec'y, 111 Broadway, New York City.
8. Electrochemical Society.	Electrochemistry and Metallurgy.	Dr. J. W. Richards, Sec'y, South Bethlehem, Pa.

* The corresponding establishment in Great Britain is The National Physical Laboratory.

Each of the above societies is broadly concerned with the scientific, engineering, economic and ethical aspects of its particular field. Every engineer should belong to one or more of these societies. They are helpful to their members, and they serve to bring the much needed counsel of the scientist and the engineer effectively into municipal, state and national affairs.

All of these societies issue important publications, and all, or nearly all, of them make special provision for student members. A letter addressed to the headquarters office of any of the societies will bring full information concerning the society.

MAGNETIC AND ELECTRIC UNITS AND SYMBOLS.

(Used in this text.)

Centimeter-gram-second system.	*Ampere-ohm-volt system.*
m = pole strength. *Unit pole* is defined in Art. 4.	There are no recognized names for the units of m, H and Φ in the ampere-ohm-volt system. Hence all quantities should be expressed in c.g.s. units in any equation containing m or H or Φ.
H = field intensity. The *gauss* is defined in Art. 10. *Note.* H is sometimes used to represent a quantity of heat.	
Φ = magnetic flux. The *maxwell* or *line* is defined in Art. 12.	
i *or* I = current. The *abampere* is defined in Art. 21.	The *ampere* is defined as one tenth of an abampere. The legal definition of the ampere is given in Art. 24. One ampere = 0.1 abampere.
r or R = resistance. A portion of a circuit has a resistance of one *abohm* when one erg of heat is developed in it by one abampere in one second.	A portion of a circuit has a resistance of one *ohm* when one joule of heat is developed in it by one ampere in one second. One ohm = 10^9 abohms.
e or E = electromotive force. The *abvolt* is the electromotive force between the terminals of a resistance of one abohm when a current of one abampere is flowing through it.	The *volt* is the electromotive force between the terminals of a resistance of one ohm when a current of one ampere is flowing through it. One volt = 10^8 abvolts.
L = inductance. A circuit has an inductance of one *abhenry* when one abvolt causes the current in the circuit to increase at the rate of one abampere per second.	A circuit has an inductance of one *henry* when one volt causes the current in the circuit to increase at the rate of one ampere per second. One henry = 10^9 abhenrys.
q or Q = electric charge. The *abcoulomb* is the charge carried in one second by one abampere.	The *coulomb* is the charge carried in one second by one ampere. One coulomb = 0.1 abcoulomb.

C = capacity. A condenser has a capacity of one *abfarad* when one abcoulomb of charge is drawn out of one plate of the condenser and pushed into the other plate by an electromotive force of one abvolt.

A condenser has a capacity of one *farad* when one coulomb of charge is drawn out of one plate of the condenser and pushed into the other plate by an electromotive force of one volt.

One farad = 10^{-9} abfarads.

THE "ELECTROSTATIC" SYSTEM OF C.G.S. UNITS.

(Not used in this text.)

The *statcoulomb* is the "electrostatic" unit of charge; it is a charge which will repel an equal charge with a force of one dyne at a distance of one centimeter in air. See Art. 69 of Chapter V and see Problem 115 on page 140. One statcoulomb is equal to 3.33 ($\times 10^{-11}$) abcoulomb.

The *statampere* is the flow of one statcoulomb per second.

The *statvolt* is an electromotive force which will do work at the rate of one erg per second when "propelling" a current of one statampere. One statvolt is equal to 3×10^{10} abvolts.

The *statohm* is a resistance in which one erg per second of heat is developed by a current of one statampere.

The *statfarad* is the "electrostatic" unit of condenser capacity. A condenser has a capacity of one statfarad when one statvolt will draw one statcoulomb out of one plate and force into it the other plate of the condenser

The *stathenry* is the "electrostatic" unit of inductance. A circuit has an inductance of one stathenry when one statvolt will cause a current in the circuit to increase at the rate of one statampere per second.

The *statgauss* is the "electrostatic" unit of magnetic field intensity. A magnetic field has an intensity of one statgauss when it will push sidewise with a force of one dyne on one centimeter of wire carrying a current of one statampere, the wire being at right angles to the field. Etc. Etc. Etc.

A very full discussion of the two systems of electric and magnetic units is given on pages 240–253 of Nichols and Franklin's *Elements of Physics*, Vol. II.

CONTENTS.

LEGAL STANDARDS AND PRACTICAL DEFINITIONS OF UNITS.

The definitions given or referred to on pages xiii and xiv are the scientifically correct* definitions of *m*, *H*, *i*, *r*, *e*, etc.; but for the engineer everything relating to electrical and magnetic units must be based on the legal standards; see pages 220 and 221. Thus the engineer's definition of the gauss may be based on the legal standard ampere, using equation (6) on page 29; or the engineer may define the unit of magnetic flux (the maxwell) as the amount of flux which must be cut per second to give an induced electromotive force of one abvolt (one hundred-millionth of a volt), in accordance with equation (16) on page 80, and the gauss may be then defined as one maxwell per square centimeter in accordance with equation (4) on page 16.

* Some variations in these definitions may be made without departing from the historical and scientific point of view.

CHAPTER I.

MAGNETISM AND THE MAGNETIC EFFECT OF THE ELECTRIC CURRENT.

1. **Preliminary statements concerning the electric current and the electric circuit.**—Because of the almost universal use of the familiar dry cell or dry battery for operating electric bells and flash lamps, and because of the supply of electric current nearly everywhere for lighting and power, we may, in beginning our study of electricity and magnetism, take the source of electric current for granted and devote our attention to the properties of the electric current; and from the point of view of elementary theory we are concerned almost wholly with three groups of effects, namely, (*a*) The magnetic effect of the electric current, (*b*) The chemical effect of the electric current, and (*c*) The heating effect of the electric current.

One aspect of the magnetic effect of the electric current is exemplified by the *electro-magnet* which consists of a winding of insulated wire on an iron rod or core. When the winding of wire is connected to a battery the iron core attracts other pieces of iron and is said to be *magnetized*. When the winding of wire is disconnected from the battery the iron rod loses its magnetism. A rod of hardened steel may be magnetized in the same way, but a rod of hardened steel retains its magnetism more or less persistently when it is removed from the winding of wire or when the winding or wire is disconnected from the battery. Such a magnetized rod of hardened steel is called a *permanent magnet*, or, simply, a *magnet*. The magnetic effect of the electric current exhibits itself in a variety of ways. Thus the action of the dynamo as an electric generator is one aspect of the magnetic effect of the electric current.

One aspect of the chemical effect of the electric current is exem-

plified by the operation of electro-plating. Thus Fig. 1 shows a battery connected to two strips of copper C and T both of which dip into a solution of copper sulphate. Under these con-

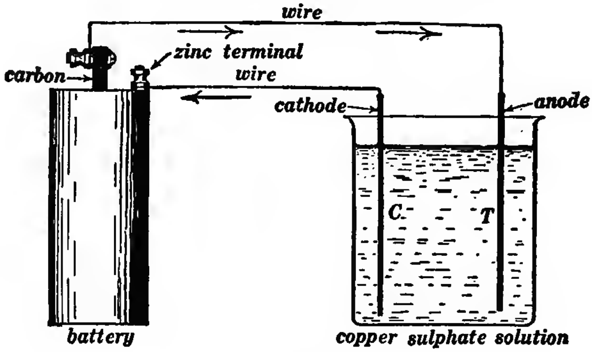

Fig. 1.

ditions metallic copper is deposited on C and copper is dissolved off T. Another aspect of the chemical effect of the electric current is exemplified by an ordinary battery which is an arrangement in which the energy of chemical action maintains a current. See Art. 25.

The heating effect of the electric current is exemplified by the familiar flash lamp the essential features of which are shown in

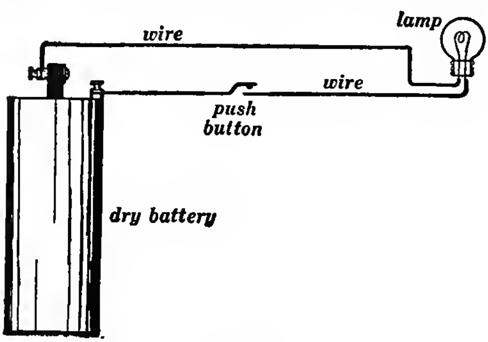

Fig. 2.

Fig. 2. The lamp is a piece of very fine tungsten wire mounted in a glass bulb.

The electric current and the electric circuit.—When the above described effects are produced an *electric current* is said to *flow* through the wire. The production of an electric current always requires an *electric generator* such as a battery or dynamo. The path of the current is usually a wire, and this path is called the *electric circuit.* A steady electric current always flows through a *complete circuit*, that is to say, through a circuit which goes out from one terminal of a battery (or dynamo) and returns to the other terminal of the battery (or dynamo) without a break. Such a circuit is called a *closed circuit.* When the circuit is not complete it is said to be an *open circuit.* The electric current ceases to flow through a circuit when the circuit is opened or broken.

An electric current which lasts for a very short time, a few thousandths of a second, for example, can flow in an incomplete or open circuit. In such a case very important effects are observed at the place where the circuit is broken. These effects, which we may very properly call **gap effects**, are discussed in chapters V, VI and VII.

Conductors and insulators.—The carbon plate of the battery forms a portion of the electric circuit in Figs. 1 and 2, the wire forms a portion of the circuit in Figs. 1 and 2, and the solution of copper sulphate forms a portion of the circuit in Fig. 1. Any substance which can serve as a portion of an electric circuit (any substance through which the electric current can "flow"), is called an *electrical conductor.* Thus metals, carbon and salt and acid solutions are electrical conductors. Many substances, such as dry wood, rubber, glass and air, cannot* serve as portions of an electric cirucit at ordinary temperatures, that is to say, the electric current cannot "flow" through such substances to any appreciable extent, and such substances are called *insulators.*

* This statement is not strictly true; what is called an insulator is merely a very poor conductor.

PROBLEMS.

1. The accompanying diagram Fig. 3 shows a lamp L with connections arranged so that the lamp can be turned on or off at switch A (or B) regardless of how switch B (or A) stands. Make four diagrams like Fig. 3 showing the four possible com-

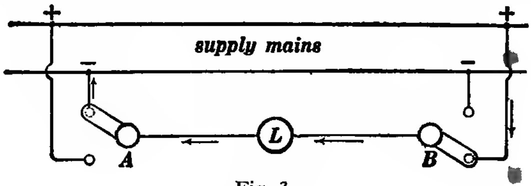

Fig. 3.

binations of switch-positions, and indicate the flow of current, if any, by arrows in each case.

2. The six small circles in Fig. 4 represent the contact posts on a double-pole, double-throw switch, and the dotted lines

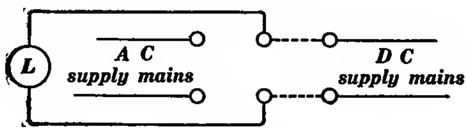

Fig. 4.

represent the switch blades. The diagram shows the lamp L taking current from the direct-current mains. Make a diagram showing the lamp taking current from the alternating current mains.

3. The six small circles in Fig. 5 represent the contact posts on a double-pole, double-throw switch with crossed connections

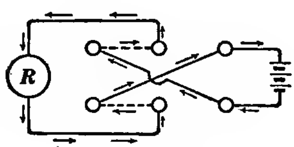

Fig. 5.

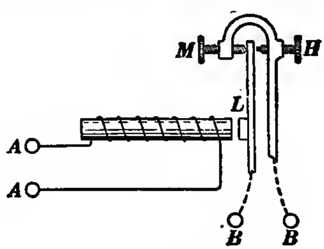

Fig. 6.

adapting it for use as a reversing switch, and the dotted lines represent the switch blades. Make a diagram showing a reversed flow of current through the receiving circuit *R*.

4. Figure 6 shows the diagram of connections of an ordinary telegraph relay, a "local" circuit connected to the binding posts *BB* is opened and closed as the lever *L* of the relay is moved back and forth by pulses of current coming over the telegraph line which is connected through the binding posts *AA* to ground. *M* is a screw with a metal tip, and *H* is a screw with a hard-rubber insulating tip . Make a diagram showing *M* and *H* interchanged, and showing *AA* and *BB* connected to each other and to a battery so that the relay will buzz like an ordinary interrupter bell.

5. It is possible to connect any number of bell circuits to one battery, and have any number of push buttons arranged to close each bell circuit independently. Make a diagram showing three bells connected to a single battery with two push buttons arranged to close the circuit of each bell independently.

6. Suppose an electric bell is to ring so as to give a starting signal to the motor-man on a 4-car train, and suppose that the bell is to ring only when every one of the four gate-men pushes a contact button, four contact buttons in all. Make a diagram of the connections.

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2. **Poles of a magnet.**—The familiar property of a magnet, namely, its attraction for iron, is possessed only by certain parts of the magnet. These parts of a magnet are called the *poles of the magnet*. For example, the poles of a straight bar-magnet are usually at the ends of the bar. Thus Fig. 7 shows the appearance of a bar-magnet which has been dipped into iron filings. The filings cling chiefly to the ends of the magnet.

When a bar-magnet is suspended in a horizontal position by a fine thread, it places itself approximately north and south like a compass needle. The north pointing end of the magnet is

called its *north pole*, and the south pointing end of the magnet is called its *south pole*.

The north poles of two magnets repel each other, the south poles of two magnets repel each other, and the north pole of one magnet attracts the south pole of another magnet; that is to say, *like magnetic poles repel each other, and unlike magnetic poles attract each other.*

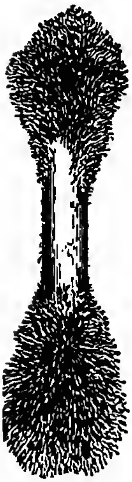

Fig. 7.

The mutual force action of two magnets is, in general, resolvable into four parts, namely, the forces with which the respective poles of one magnet attract or repel the respective poles of the other magnet. *In the following discussion we consider only the force with which one pole of a magnet acts upon one pole of another magnet*, not the forces with which one complete magnet acts on another complete magnet.

3. Distributed poles and concentrated poles.—The poles of a bar magnet are always distributed over considerable portions of the bar. This is especially the case with short thick bars. In the case of a long slim bar magnet, however, the poles are ordinarily approximately concentrated at the ends of the bar. The forces of attraction and repulsion of concentrated magnet poles are easily formulated, therefore *the following discussion applies to ideally concentrated poles at the ends of ideally slim bar magnets.*

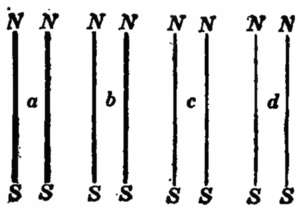

Fig. 8.

Pairs of exactly similar magnets.

4. Definition of unit pole.—Consider a large number of pairs of magnets *a b, c, d,* etc., as shown in Fig. 8, the two magnets of each pair being exactly alike.* From such a set

* That is, the magnets of each pair are made of identically the same kind of steel, subjected to the same kind of heat treatment and magnetized by the same means.

it would be possible to select a pair of magnets such that the north pole of one magnet would repel the north pole of the other with a force of one dyne when they (the two north poles) are one centimeter apart; each pole of such a pair is called a *unit pole.* That is, *a unit pole is a pole which will exert a force of one dyne upon another unit pole at a distance of one centimeter.*

5. **Strength of pole.**—Let us choose a slim magnet with unit poles, and let us use one of these unit poles as a "test pole."

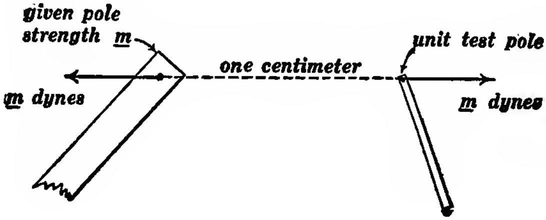

Fig. 9.

Any given magnet pole is said to have more or less *strength* according as it exerts more or less force on our "test pole" at a given distance. And the force m (in dynes) with which the given pole attracts or repels (or is attracted or repelled by) the unit test pole at a distance of one centimeter is taken as the measure of the strength of the given pole. That is, *a given pole has m units of strength when it will exert a force of m dynes on a unit pole at a distance of one centimeter,* as indicated in Fig. 9.

6. **Attraction and repulsion of magnet poles.**—Unlike poles attract and like poles repel each other, as stated in Art. 2. When the two attracting or repelling poles are *unit poles* their attraction or repulsion is equal to one dyne when they are one centimeter apart, and the attraction or repulsion of two poles whose respective strengths are m' and m'' is equal to $m'm''$ dynes when the poles are one centimeter apart. *One may think*

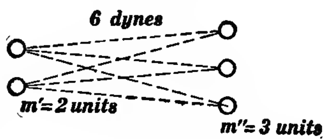

Fig. 10.

of each unit *of* m' *as exerting a force of one dyne on each* unit *of* m''. Thus if $m' = 3$ units and $m'' = 2$ units, then the force of attraction or repulsion will be six dynes, as indicated in Fig. 10, where each dotted line represents one dyne.

7. Coulomb's law. Complete expression for the force of attraction or repulsion of two magnet poles.—Coulomb discovered in 1800 that the force of attraction or repulsion of two magnet poles is inversely proportional to the square of the distance between them. But the force of attraction or repulsion of two magnet poles when they are one centimeter apart is $m'm''$ dynes as explained in Art. 6. Therefore, according to Coulomb's law, the force of attraction or repulsion is $\frac{m'm''}{r^2}$ dynes when the poles are r centimeters apart. That is:

$$F = \frac{m'm''}{r^2} \tag{1}$$

in which m' and m'' are the respective strengths of two magnet poles, r is their distance apart in centimeters, and F is the force in dynes with which the poles attract or repel each other.

8. Equation (1) is really a differential equation.—Any physical thing must be to some extent idealized if it is to be formulated mathematically, and, as a rule, this idealization must be excessive if the mathematics is to be simple. The elementary theory of magnetism is a remarkable example of excessive idealization; but without this excessive idealization the mathematics is much too complicated for the beginner.*

* Many attempts have been made to develope the elementary theory of magnetism without excessive idealization, and many of these attempts have been made by men who do not understand *divergence* and *curl*. Space distributions, like magnetic field and electric field, are sometimes conditioned by divergence and sometimes conditioned by curl, and the conception of the concentrated magnet pole is artificial not merely because it is a differential but also and chiefly because the conception of the magnetic pole grows out of the attempt to use the simpler idea of divergence in place of the more difficult idea of curl. The mathematical physicist recognizes the artificial character of the elementary theory of magnetism, and he also understands it! and he knows what the alternative is, namely, real, downright mathematics.

Fig. 11 represents two indefinitely broad and indefinitely long strips or ribbons of sheet steel MM and $M'M'$; NN is the edge of one ribbon, SS is the edge of the other ribbon, and the ribbons are supposed to be magnetized so that a north pole is spread along NN (a units of pole per centimeter of length) and a south pole is

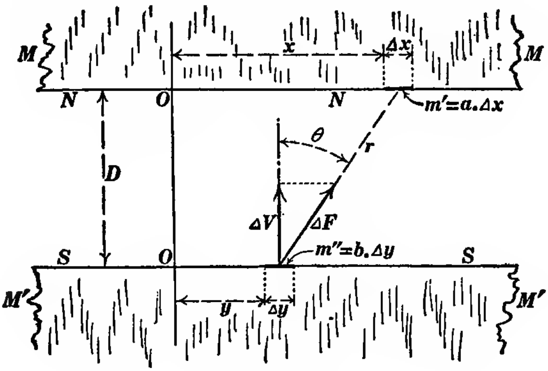

Fig. 11.

spread along SS (b units of pole per centimeter of length). It is required to find the total force V with which the pole NN pulls straight upwards on SS.

Consider the two elements Δx and Δy. The element Δx is a concentrated pole $m' = a.\Delta x$; the element Δy is a concentrated pole $m'' = b.\Delta y$; the force ΔF exerted on Δy by Δx is $\Delta F = \dfrac{a.\Delta x \times b.\Delta y}{(x - y)^2 + D^2}$, according to equation (1); and the vertical component of ΔF is $\Delta V = \Delta F \times \cos\theta = \Delta F . \dfrac{D}{[(x - y)^2 + D^2]^{1/2}}$.

Therefore

$$\Delta V = \frac{abD}{[(x - y)^2 + D^2]^{3/2}}.\Delta x.\Delta y$$

and the total force V acting on SS is to be found by integrating this expression. This integration, however, involves too much algebraic manipulation to be given here.

The ideal concentrated pole is really a differential, and the correct definition of dy, for example, is that dy is the amount that y would increase during a finite time dt if the rate of increase of y were to remain constantly at the value it had at the start, at the instant t. It is almost beyond the power of the English language to apply this entirely correct definition of a differential to such a concept as the ideal concentrated pole, and if the correct mathematical definition of an ideal concentrated pole were to be formulated, the conditional terms "would increase" "if" and "were to remain" would make it entirely useless as a practicable form of thought. It is much better to think of an ideal concentrated pole as an infinitely small pole! even if such a thing is, strictly speaking, unthinkable! These are the considerations, more or less Irish, to be sure, which lead the engineer and the

mathematical physicist to an attitude of contempt for the purist in mathematics who insists on the strictly correct definition of the differential!

PROBLEMS.

7. Figure 12 represents two similar bar magnets, the poles being assumed to be concentrated at the extreme ends of the bars and the strength of each pole being 350 units. Find the total force exerted on one magnet by the other.

Fig. 12.

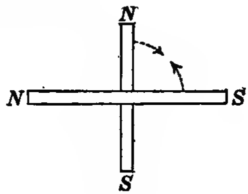

Fig. 13.

8. The two magnets of the previous problem are arranged as shown in Fig. 13. Find the total force action exerted on one magnet by the other.

9. **The magnetic field.**—When iron filings are dusted over a pane of glass which lies flat on a magnet, the filings arrange themselves in regular filaments as shown in Figs. 14 and 15, if the glass plate is jarred slightly. Figure 14 shows the filaments

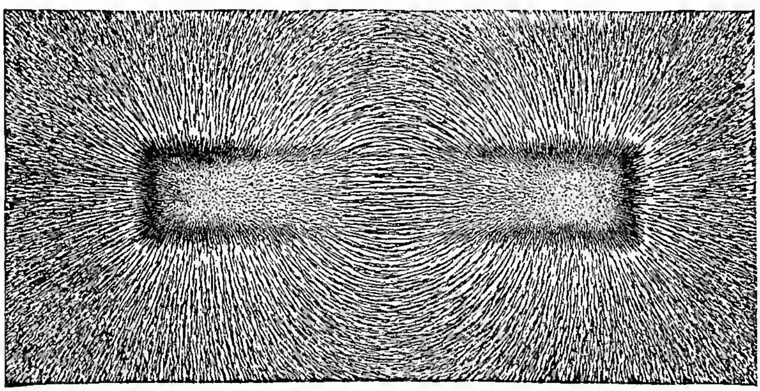

Fig. 14.

of iron filings in the neighborhood of a single bar magnet, and Fig. 15 shows the filaments of filings between the unlike poles of two large magnets.

The region surrounding a magnet is called a *magnetic field*, and the filaments of iron filings in Figs. 14 and 15 show the trend of what are called the *lines of force* of the magnetic field. *Indeed any region is a magnetic field in which a suspended* magnetic needle (like a compass needle) points in a definite direction*, and the

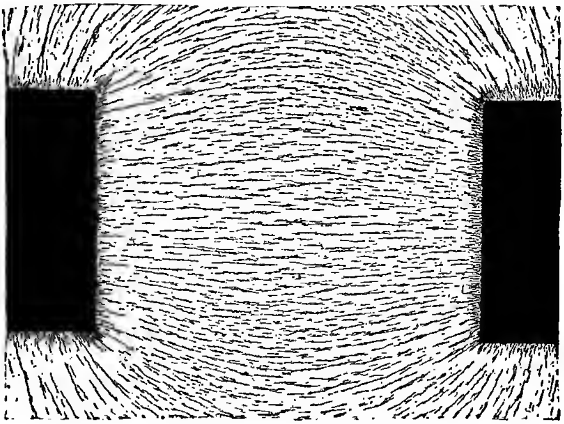

Fig. 15.

direction in which the north pole of the needle points is called the *direction of the magnetic field* at the place where the needle is suspended.

10. Intensity of a magnetic field at a point.—When a magnet is placed in a magnetic field a force is exerted on each pole of the magnet by the field. Thus the two arrows in Fig. 16 represent the forces which are exerted on the poles of a small magnet which is placed in the magnetic field between two large magnet poles (see Fig. 15).

The force H *in dynes which a magnetic field exerts on a "unit test pole" is used as a measure of the strength or intensity of the*

* The needle is supposed to be suspended at its center of gravity.

field, and this **force-per-unit-pole** *is hereafter spoken of simply as the* **intensity** *of the field.* The unit of magnetic field intensity (one dyne-per-unit-pole) is called the *gauss.* That is to say, a

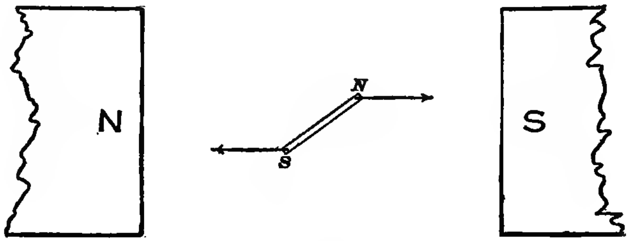

Fig. 16.

The arrows show the forces which act upon the poles of the small magnet.

magnetic field has an intensity of one gauss when it will exert a force of one dyne upon a unit pole.

Complete expression for the force exerted on a magnet pole by a magnetic field.—A magnetic field of which the intensity is H gausses exerts a force of H dynes upon a unit pole as above explained, and it exerts a force of mH dynes upon a pole of which the strength is is m units. That is:

$$F = mH \tag{2}$$

in which F is the force in dynes which is exerted on a pole of strength m by a field of intensity H.

Uniform and non-uniform fields.—A magnetic field is said to be *uniform* when it has everywhere the same direction and the same intensity, otherwise the field is said to be *non-uniform.* The earth's magnetic field is sensibly uniform throughout a room. The magnetic field surrounding a magnet is non-uniform. The magnetic field surrounding an electric wire is non-uniform.

The oscillating magnet.—A suspended magnet points magnetic north and south. If turned through an angle θ, the suspended magnet is acted upon by a torque $T = -mlH \sin \theta$, or, if θ is

small, $T = -mlH \cdot \theta$, where θ is expressed in radians. Therefore the "stiffness coefficient" of the suspended magnet is $b = mlH$, and, as explained in Art. 57 of *Lessons in Mechanics*, we have

$$4\pi^2 n^2 K = mlH$$

where n is the number of oscillations per second of the suspended magnet, K is the moment of inertia of the suspended magnet referred to the axis of suspension, l is the length of the magnet (distance between its poles), $\pm m$ is the strength of the poles of the magnet, and H is the horizontal component of the earth's magnetic field.

PROBLEMS.

9. A bar magnet 30 cm. × 1 cm. × 1 cm. weighing 235 grams is balanced horizontally on a knife edge at a place where the horizontal component of the earth's magnetic field is 0.18 gauss and the vertical component of the earth's magnet field is 0.48 gauss. The poles of the magnet are assumed to be concentrated at the extreme ends of the bar and the strength of the poles is ± 700 units. Find the horizontal distance of the knife edge from the middle of the bar, taking the acceleration of gravity to be 980 centimeters per second per second.

10. The magnet of the previous problem is suspended so as to stand horizontally and oscillate about a vertical axis. Find the number of complete oscillations per second.

11. Direction and intensity of the magnetic field surrounding an "isolated" magnet pole of strength M.—By an "isolated" magnet pole is meant one pole of a very long slim magnet—the other pole being so far away as to be negligible in its action.

The magnetic field in the neighborhood of an isolated north pole is everywhere directed *away from* the pole as shown by the radiating straight lines (lines of force, as they are called) in Fig. 17. The magnetic field in the neighborhood of an isolated

south pole is everywhere directed *towards* the pole as indicated in Fig. 18.

Consider two magnet poles M and m which are r centimeters apart as shown in Fig. 19. The force F with which M repels

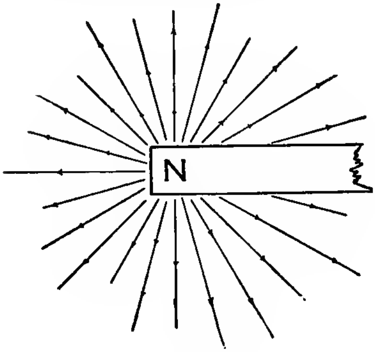

Fig. 17.

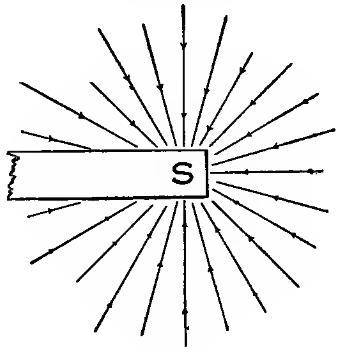

Fig. 18.

m is equal to Mm/r^2 according to Art. 7; but the force which is exerted on m may also be expressed as mH, *where* H *is the intensity at* m *of the magnetic field which is due to* M. Therefore

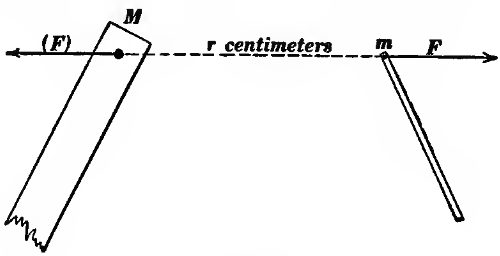

Fig. 19.

fore mH must be equal to Mm/r^2, from which we get

$$H = \frac{M}{r^2} \tag{3}$$

in which H is the intensity in gausses of the magnetic field produced by M at a place which is r centimeters from M.

PROBLEMS.

11. Given a bar magnet 30 cm. $\times$ 1 cm. $\times$ 1 cm. with poles of $\pm$ 700 units strength, and let us assume that these poles are concentrated at the extreme ends of the bar. Find the intensity of the magnetic field produced by both poles of the magnet at a point which is 18 centimeters from one pole and 24 centimeters from the other pole.

Note.—When two agencies or causes act together to produce magnetic field the intensity at any point of the field due to both causes is the vector sum of the intensities at that point due to the two causes separately.

12. The bar magnet of the previous problem is placed in an east-west position with its middle point one meter east or west of a suspended magnetic needle. The needle points due north before the large magnet is placed in position, find the angle through which the needle is turned by the large magnet, horizontal component of the earth's magnetic field being 0.18 gauss.

Note.—The terms north and south, and east and west as used in this problem refer to magnetic north and south and magnetic east and west.

13. A bar magnet 1 cm. $\times$ 1 cm. $\times$ 30 cm. whose mass is 235 grams deflects a suspended magnet needle through an angle of $13\frac{1}{2}$ degrees when arranged as described in problem 12; and when suspended and set oscillating as specified in problem 10, the magnet makes 15 complete oscillations in 102 seconds. Calculate the value of the horizontal component of the earth's magnetic field and calculate the strength of each pole of the magnet.

Note.—This problem illustrates the essentials of Gauss's method for measuring the horizontal component of the earth's magnetic field and the strength of the poles of a magnet. The distance between the poles is assumed to be 30 centimeters in the solution of the problem whereas it is in fact considerably less than 30 centimeters. Gauss's method as carried out in magnetic surveys or in the laboratory is arranged so as to eliminate this source of error. See Art. 38 of Appendix *C*. See also Kohrausch's *Physical Measurements*, pages 240–245; London, J. & A. Churchill, 1894.

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12. Definition of magnetic flux.—Figure 20 represents a plane

area of a square centimeters at right angles to a uniform magnetic field of which the intensity is H gausses. The product aH is called the *magnetic flux* across the area. That is

$$\Phi = aH \tag{4}$$

where Φ is the magnetic flux across an area of a square centimeters at right angles to a magnetic field of which the intensity is H gausses.

The unit of magnetic flux is the flux across an area of one square centimeter $(a = 1)$ when the area is at right angles to a magnetic field of which the intensity is one gauss $(H = 1)$, or it is the flux across n square centimeters of area when the area is at right angles to a magnetic field of which the intensity is $1/n$th of a gauss. The unit of magnetic flux is properly called the *maxwell;* **the common usage however is to call the unit of flux** *the line of force* **for the following reason:**

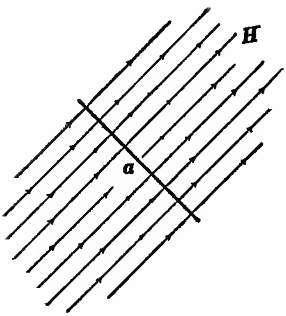


Fig. 20.

Consider any magnetic field whatever, and imagine a surface or shell BB which is everywhere at right angles to the field as shown in Fig. 21. Imagine the surface BB to be divided up into

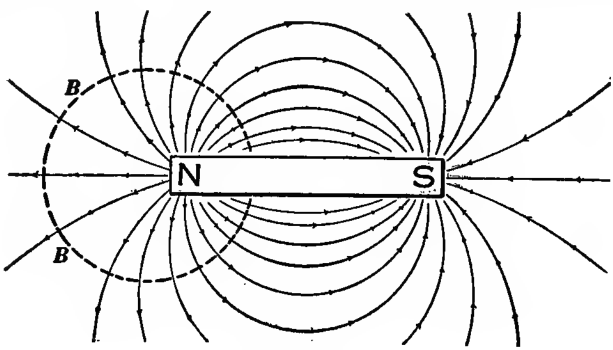


Fig. 21.

small squares such that one unit of magnetic flux crosses each square, and imagine lines of force to be drawn so that one line of force passes through each square. *Then the magnetic flux across any other surface or shell* AA *placed anywhere in the field will be equal to the number of these* **lines of force** *which cross* AA.

Proposition.—The magnetic flux Φ which emanates from a north pole of strength M is

$$\Phi = 4\pi M \tag{5}$$

Imagine a sphere of radius r to be drawn with its center at the pole M in Fig. 19. The magnetic field due to M is everywhere at right angles to this spherical surface and its intensity is M/r^2 at the spherical surface. Therefore we must multiply the intensity M/r^2 by the area $4\pi r^2$ of the sphere to get the flux Φ, so that $\Phi = 4\pi M$.

In this argument we have neglected the small part of the spherical surface where the steel bar passes through it. An amount of flux $\Phi = 4\pi M$ *emanates from* a north pole or *comes in to* a south pole.

13. **Tension of the magnetic field.**—The force F in dynes with which the two broad flat magnet poles NN and SS in Fig. 22 are drawn towards each other is

$$F = \frac{s}{8\pi} \cdot H^2 \tag{i}$$

where s is the area in square centimeters of each pole face in

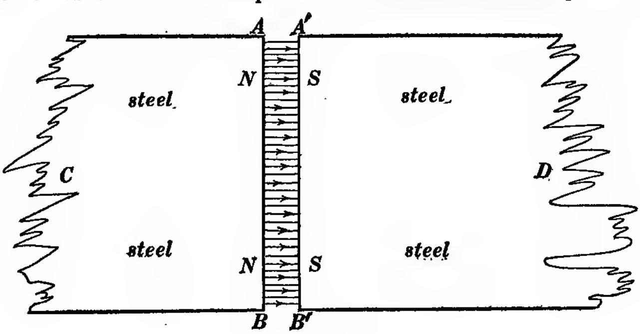


Fig. 22.

Fig. 22, and H is the intensity in gausses of the approximately uniform magnetic field in the air gap between the pole faces. The attraction of NN and SS in Fig. 22 shows that the inter-

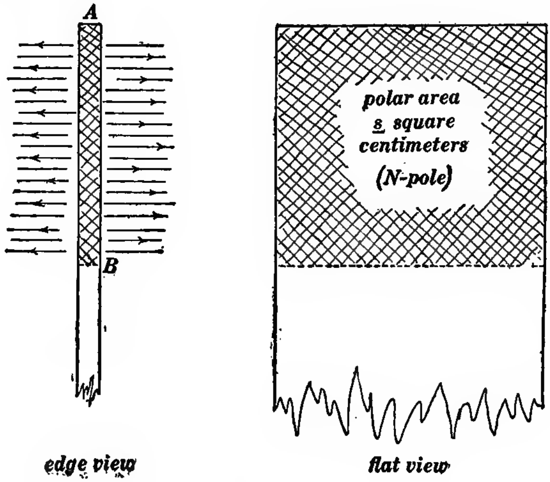


Fig. 23.

vening region is in a state of tension, and the value of this tension in dynes per square centimeter is equal to F/s or to $H^2/8\pi$.

As used by the electrical engineer equation (i) is ordinarily somewhat modified as follows: The total magnetic flux Φ which crosses from pole face to pole face in Fig. 22 is $\Phi = sH$ according to equation (4) of Art. 12 and therefore $H = \Phi/s$. Substituting this value of H in equation (i), we get

$$F(\text{in dynes}) = \frac{1}{8\pi} \cdot \frac{\Phi^2}{s} \tag{ii}$$

The straightforward proof of equation (i) as given by Maxwell is far beyond the undergraduate, and the following derivation of equation (i) is given chiefly to serve as an example of the simplification of the mathematics of a problem by considering idealized conditions, and very certainly the two broad flat magnet poles in Fig. 24 are idealized.

Figure 23 shows one end of a very broad flat piece of thin steel magnetized so as to have a north pole spread uniformly over a large portion of one end. Let m be the total strength of this pole. Then $4\pi m$ is the total amount of flux emanating from the pole *and this flux emanates as a uniform magnetic field on both sides of the flat pole* as indicated in the edge view in Fig. 23. Let H_1 be the intensity of this field on either side, then $H_1 s$ is the magnetic flux emanating on either side, and $2H_1 s$ is the total magnetic flux emanating from the pole. Therefore $2H_1 s = 4\pi m$ so that

$$H_1 = \frac{2\pi m}{s} \qquad \text{(iii)}$$

Figure 24 shows two poles like Fig. 23 placed side by side, a north pole and a south pole; each pole m being in the field of strength $\frac{2\pi m}{s}$ which is due to the other, and therefore each pole is acted upon by a force $m \times \frac{2\pi m}{s}$ drawing it towards the other pole according to equation (2). That is to say, the two poles in Fig. 24 are drawn towards each other by a force

$$F = \frac{2\pi m^2}{s} \qquad \text{(iv)}$$

In the region beween the poles in Fig. 24 the field intensities due to the separate or individual poles are in the *same* direction so that the actual intensity of the field between the two poles is $H = 2H_1$ so that

$$H = \frac{4\pi m}{s} \qquad \text{(v)}$$

But in each of the regions RR and $R'R'$ the field intensities due to the separate or individual poles are *opposite* in direction so that no field at all exists in these regions The only magnetic field in Fig. 24 is the field between the poles, and the force in equation (iv) is due to the tension of this field. Substituting the value of m from (v) in (iv) we get $F = \frac{s}{8\pi}.H^2$.

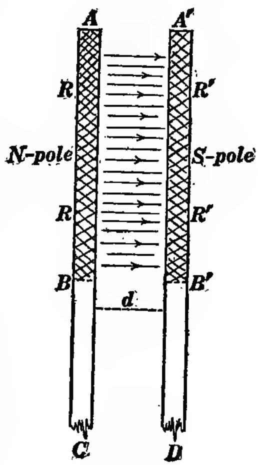


Fig. 24.

Remark.—Equations (i) and (ii) are strictly applicable to Fig. 24, but they give a force which is a little in excess of the true force which draws the two pieces of steel together in Fig. 22, because the entire tension of the field in Fig. 22 does not act on the steel bars.

PROBLEMS.

14. The pole face of the field magnet of a dynamo has an area 20 centimeters by 30 centimeters. The magnetic field between the pole faces and the armature core is perpendicular to the pole face at each point and its intensity is 6,000 gausses. Calculate

the number of lines of force which pass from the pole face into the armature core.

15. A steel bar magnet 2 square centimeters in sectional area has poles at its ends and the strength of the poles is + 1500 units and − 1500 units. The bar is cut in two at the middle (before being magnetized, of course) and the two halves are accurately faced so that they fit and thus give practically a continuous bar. How much force would be required to pull the two halves apart?

14. Oersted's experiment. The right-handed screw convention as to the direction of current.—Figure 25*a* represents a top view of a magnetic compass with a wire beneath. When no current flows through the wire the compass needle stands parallel to the wire, let us say; then when the wire is connected to a battery the compass needle turns to the indicated position (or it may turn in the opposite direction). This effect was discovered by the Danish physicist Oersted in 1819, the first discovery relating to the magnetic effect of the electric current.

Fixing the attention on the north pole of the compass needle in Fig. 25*a*, it may be stated that *the north pole starts to circle round the wire* (which it cannot actually do because of the sup-

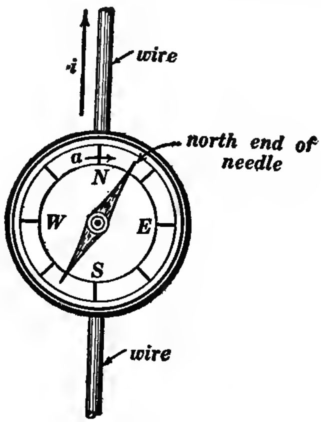


Fig. 25*a*.

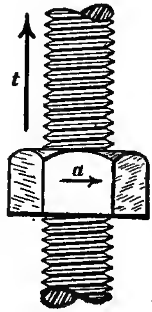


Fig. 25*b*

porting pivot and because of the force exerted on the south pole of the needle), *and* **the direction of the current in the wire is thought of as the direction a nut on a right-handed screw would travel if turned in the direction in which the north pole of the compass needle starts to circle round the wire.** The exact meaning of this conventional rule may be understood by comparing Figs. 25*a* and 25*b*.

When an iron rod is magnetized by the flow of current round it, *the north pole of the rod is at the end towards which a nut would travel* (*on a right-handed screw*) *if the nut were turned in the direction in which the current flows round the rod* as shown in Fig. 26.

When it is desired to show an end-view of a wire through which current is flowing, the section of the wire is represented by a small circle, *current flowing towards the reader is represented by a dot in the circle*, **as if one were looking endwise at the point of an arrow**; and *current flowing away from the reader is represented by a cross in the circle*, **as if one were looking at the feathered end of an arrow,** as shown in Fig. 27.

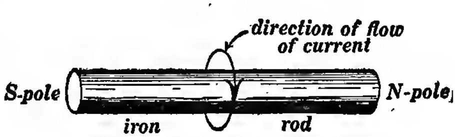


Fig. 26.


Fig. 27.

Current flowing away from reader. Current flowing towards reader.

15. **Another aspect of the magnetic effect of the electric current. Side push of the magnetic field on an electric wire.**—One aspect of the magnetic effect of the electric current is described in Art. 1; another aspect of this effect is indicated in Fig. 25*a;* and still another aspect of the effect is shown in Fig. 28. A wire *AB* through which an electric current is flowing is stretched across the end of a magnet, and the wire is pushed sidewise by the magnet as stated in the legend under the figure.

If the current is reversed or if the magnet is turned end for end the side push on the wire is reversed.*

The side force on the wire in Fig. 28 is exerted by the magnet, and this force is no doubt transmitted by something which connects the magnet and the wire together, namely, the *magnetic*

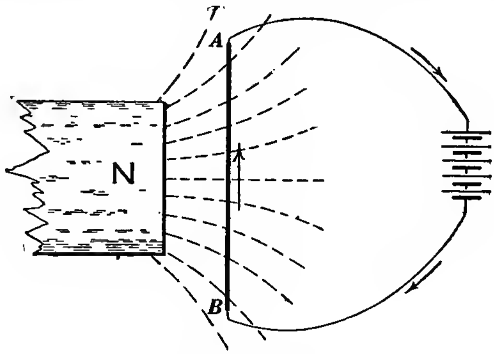


Fig. 28.
The wire *AB* is pushed away from the reader.

lines of force which emanate from the magnet. These magnetic lines of force are indicated by the dotted lines in Fig. 28.

Figure 29 shows a straight wire *AB* placed in a narrow air gap between two opposite magnet poles. The fine lines across the gap represent the magnetic lines of force in the air gap, and these lines of force push the wire sidewise (away from the reader in Fig. 29).

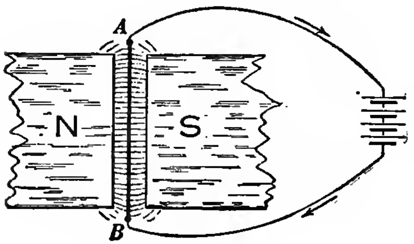


Fig. 29.
The wire *AB* is pushed away from the reader.

When an electric wire is placed in a magnetic field at right angles to the lines of force of the field, a force is exerted on the wire (a side push on the wire) at right angles to the lines of force and at right angles to the wire.

* Let it be clearly understood that the wire in Fig. 28 is neither attracted nor repelled by the magnet.

16. The magnetic field surrounding a straight electric wire.— Figure 30 is a photograph of the filaments of iron filings on a horizontal glass plate, the black circle is a hole through the plate, and a straight electric wire passes vertically through this hole. *The lines of force of the magnetic field which is produced by an electric wire encircle the wire*, as shown by the filaments of iron filings in Fig. 30.

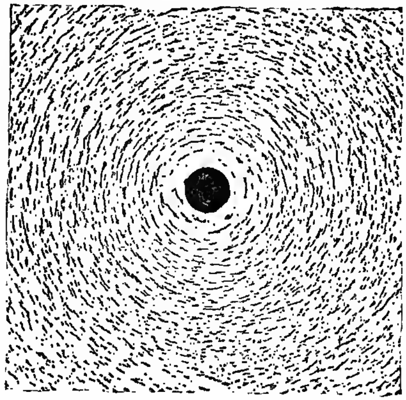


Fig. 30.

17. Explanation of the side push exerted upon an electric wire by a magnetic field.— Figure 15 represents the magnetic lines of force between two opposite magnetic poles, and the attraction of the two opposite poles for each other may be thought of as due to a state of tension in the lines of force. That is, the lines of force may be thought of as stretched, rubber-like filaments leading from pole to pole in Fig. 15, and the attraction of the two opposite

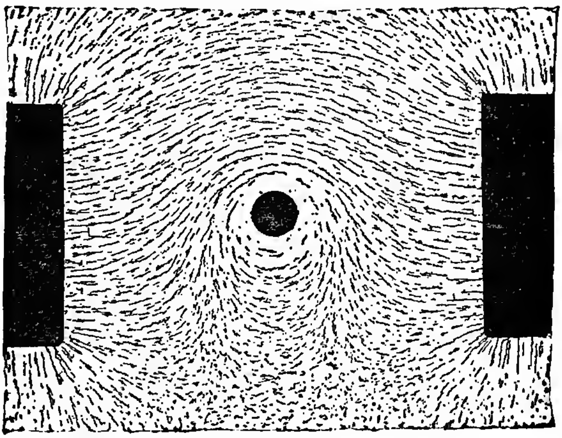


Fig. 31.

poles may be thought of as the tendency of these stretched filaments to shorten.

Figure 31 shows how the magnetic field between the two opposite poles in Fig. 15 is modified by the presence of an electric wire. The glass plate upon which the filings were dusted in Fig. 31 is horizontal, and the black circle represents a hole in the plate through which the vertical electric wire was placed. The lines of force from pole to pole pass mostly to one side of the wire in Fig. 31, and the wire is pushed sidewise by the tension of the lines for force (tendency of the lines of force to shorten).

Rule for determining direction of side push.—A wire is at right angles to a magnetic field. The direction of the field* is the direction of the force which would be exerted by the field on a north-pointing magnet pole. When current is started in the wire additional field is produced which circles round the wire in the direction in which a right-handed screw would have to be turned to travel in the direction of the current. These two fields are in the same direction on one side *a* of the wire and in opposite directions on the other side *b*, and the side push on the wire is from *a* towards *b*. This rule once it is clearly understood is very much better than the well known rule relating to thumb and two fingers of the right hand because the right-handed screw rule is consistently applicable to every aspect of the matter under consideration.

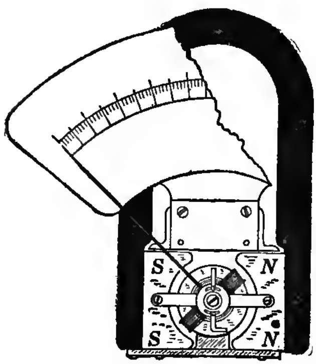


Fig. 32.

18. The direct-current ammeter.—The side push of the magnetic field on an electric wire as shown in Figs. 28 and 29, and as explained in Art. 17, is made use of in the *direct-current ammeter*, the essential features of which are shown in Figs. 32 and

* This refers, of course, to the field which exists independently of the current in the wire, the field which exists before the current starts to flow in the wire.

33. The vertical portions, or limbs, of the pivoted coil play in a narrow air space or gap space between a fixed cylinder of soft iron and the soft iron pole pieces *NN* and *SS* of a steel horse-shoe

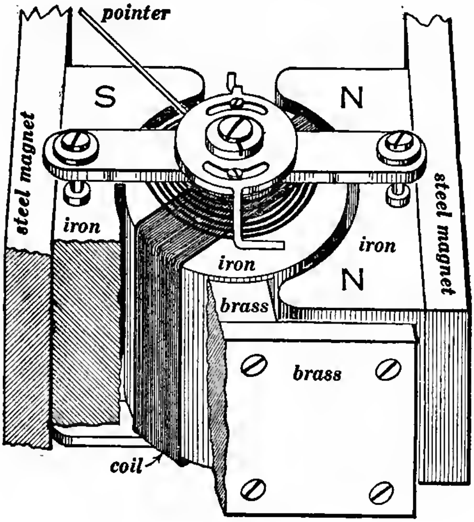


Fig. 33.

magnet. Current is led into and out of the pivoted coil by means of two hair-springs, one at each end of the pivot-axis, and the side push of the magnetic field on the limbs of the coil in the gap spaces turns the coil and moves the pointer over the scale.

19. The magnetic blow-out.—The side push of the magnetic field upon the carrier of an electric current, as shown in Figs. 28 and 29, and as explained in Art 17, is made use of in the magnetic blow-out.

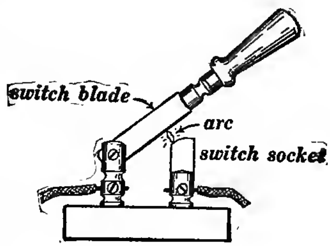


Fig. 34.

When a electric switch is opened the current continues for a short time to flow across the opening, forming what is called an electric arc, as shown in Fig. 34. This arc melts the contact parts of the switch, and the switch is soon

spoiled. This difficulty may be obviated to some extent by always opening the switch quickly and unhesitatingly, but where a switch is to be opened and closed hundreds of times per day, as in the control of a street car motor, it is necessary to blow out the arc so as to avoid the rapid wear of the switch contacts by fusion. This blowing out of the arc is accomplished by a magnet placed as shown in Fig. 35. This magnet pushes sidewise on the arc (towards or away from the reader in Fig. 35), and this sidewise push on the arc lengthens it very quickly and breaks the circuit.

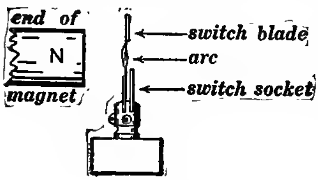


Fig. 35.
The arc is pushed towards or away from the reader.

20. The electric motor (direct-current type).—The side push of a magnetic field on an electric wire, as shown in Figs. 28 and 29 and as explained in Art. 17, is made use of in the electric motor. The following discussion is intended to explain only the essential principles and to describe only the essential features of the direct-current motor, the actual details of design and construction may be seen by inspecting a commercial motor.

Figure 36 shows an iron cylinder *AA* placed between the poles *N* and *S* of a powerful electromagnet, the *field magnet*, as it is called. The air space or gap space between the cylinder *AA* and each pole face is an intense magnetic field as indicated by the fine lines (lines of force).

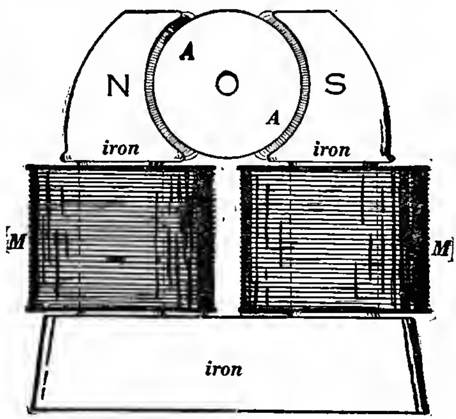


Fig. 36.

Figure 37 shows the cylinder *AA* with straight wires laid upon its surface (wires parallel to the axis of the cylinder *AA*), and the dots and crosses represent electric currents flowing towards the reader and away from the reader, respectively, as as explained in Fig. 27. *If we can arrange for the steady flow of electric current as indicated by the dots and crosses in Fig.* 37 *then*

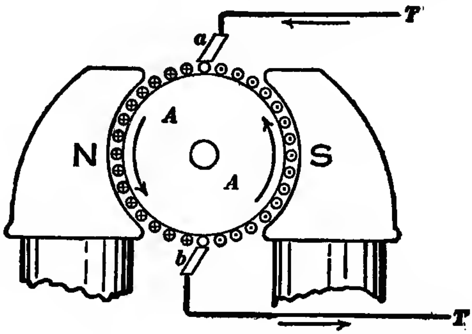


Fig. 37.

the magnetic field in the gap spaces (the fine lines of force in Fig. 36) *will push sidewise on the wires and rotate the armature in the direction of the curved arrows in Fig.* 37, **and the arrangement whereby the desired flow of current can be realized is most easily understood by considering the simplest type of armature winding, namely the RING WINDING.**

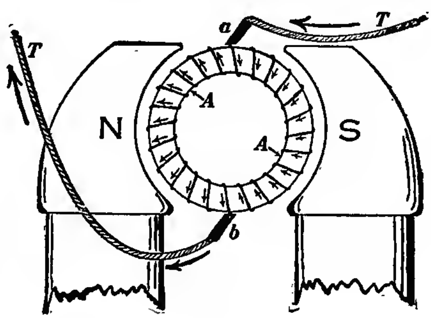


Fig. 38.

An iron ring *AA*, Fig. 38, is wound uniformly with insulated wire as shown, the ends of the wire being spliced together and soldered so as to make the winding endless. *Imagine the insulation to be removed from the outward faces of the wire windings on the ring so that two stationary metal or carbon blocks (brushes) a and b can make good electrical contact with the wire winding as*

the ring rotates. Then, if current is led into the winding through brush a and out of the winding through brush b, the current will flow towards the reader in all the wires under the S pole and away from the reader in all the wires under the N pole as indicated by the dots and crosses in Fig. 37.

In practice, short lengths of wire are attached to the various turns of wire on the ring and led to copper bars near the axis of rotation, as shown in Fig. 39. These copper bars are insulated from each other, and sliding contact is made with these copper

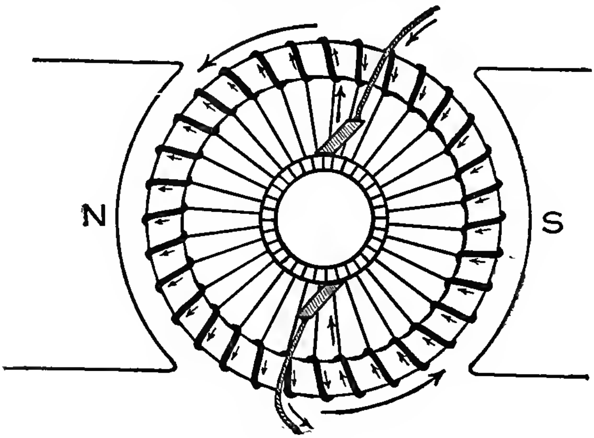


Fig. 39.

bars as indicated in Fig. 39, instead of being made as indicated in Fig. 38. The set of insulated copper bars is called the *commutator*.

The iron body of the armature (the iron ring in Figs. 38 and 39) is called the *armature core*. This core is built up of ring-shaped stampings of thin sheet iron.

The machine which is here described as the *direct-current motor* is properly called the **direct-current dynamo.** It is a **motor** when it receives electric current from some outside source and is used to drive a pump, or a lathe, or a trolly car. Exactly the same machine, when driven by a steam engine or water

wheel, can be used as an electric generator to supply direct current for driving motors or for operating electric lamps. When so used the machine is called a **dynamo electric generator** or simply a **generator.**

Remark.—Inasmuch as we are here concerned only with fundamental principles it is not necessary to refer to the *multipolar dynamo* or to the type of armature winding which is called the *drum winding,* although most direct current dynamos are nowadays of the multipolar type and have drum windings on their armatures.

21. Strength of electric current magnetically defined.—Consider a straight electric wire stretched across a uniform magnetic field, the wire being at right angles to the field as shown in Fig. 40. Let us suppose, for a moment, that the field is of

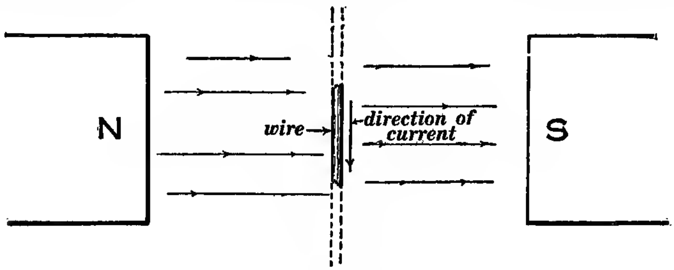


Fig. 40.
Wire is pushed towards reader.

unit intensity. The force in dynes with which this unit field pushes sidewise on one centimeter of the electric wire has been adopted as the fundamental measure of the strength of the current in the wire. This **force-per-unit-length-of-wire-per-unit-field-intensity** is called, simply, *the strength of the current in the wire,* and it is represented by the letter I. The force pushing sidewise on l centimeters of the wire is lI dynes; and if the field intensity is H gausses instead of one gauss, then the force is H times as great, or lIH dynes. That is

$$F = lIH \tag{6}$$

in which F is the force in dynes pushing sidewise on l centimeters of wire at right angles to a uniform magnetic field of which the intensity is H gausses, and I is the strength of the current in the wire.

Definition of the abampere.—A wire is said to carry a current of one abampere when one centimeter of the wire is pushed sidewise with a force of one dyne, when the wire is stretched across a magnetic field of which the intensity is one gauss, the wire being at right angles to the field. The current I in equation (6) is expressed in abamperes when F is expressed in dynes, l in centimeters and H in gausses. The abampere is the c.g.s. unit of current.

Definition of the ampere.—The ampere is defined as *one tenth of an abampere.*

22. Magnetic field due to a long straight wire carrying I abamperes of current.—If we can find the force F exerted on the magnet pole m by the wire in Fig. 41 we will know the intensity H

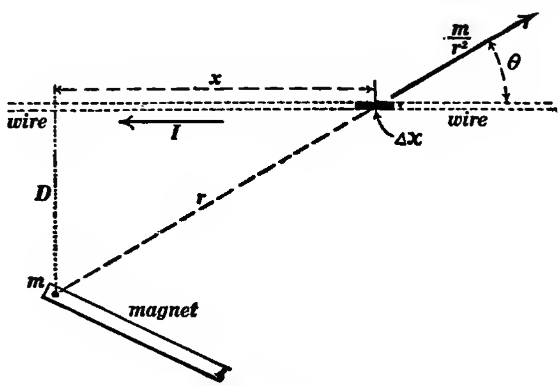


Fig. 41.

of the magnetic field at m due to the wire because $F = mH$ according to equation (2).

To find the force F let us consider the equal and opposite force F' exerted on the wire by the pole. This force F' may be calculated as follows. The field intensity at the element of wire Δx due to m is m/r^2, the component of m/r^2 which is at right angles to the wire is $\frac{m}{r^2} \sin \theta$, and the side force $\Delta F'$ (away from the reader in Fig. 41) exerted on Δx is

$$\Delta F' = \frac{mDI}{(x^2 + D^2)^{3/2}} \cdot \Delta x \tag{i}$$

according to equation (6), where $(x^2 + D^2)$ is written for r^2 and $D/\sqrt{x^2 + D^2}$ is written for $\sin \theta$. By integrating* equation (i) from $x = -\infty$ to $x = +\infty$, we get $F' = 2mI/D$, whence we get $mH = 2mI/D$ so that

$$H = \frac{2I}{D} \tag{ii}$$

where H is the intensity at m Fig. 41 of the magnetic field due to the indefinitely long wire, I is the current in the wire in abamperes and D is the distance of the point m from the axis of the wire in centimeters.

23. Magnetic field inside of a long uniformly wound coil.— A glass or fiber tube l centimeters long is wound with Z turns of insulated wire and a current of I abamperes flows through the wire. The region inside of the long tube is a uniform magnetic field, and its intensity H in gausses is

$$H = 4\pi \frac{Z}{l} \cdot I \tag{7}$$

This equation may be easily established for points along the axis of the coil by straight forward methods† but the following

* This integration involves too much algebraic manipulation to be fully explained here.

† See Franklin and MacNutt's *Advanced Electricity and Magnetism*, pages 23–25; published by Franklin and Charles, Bethlehem, Pa.

simple derivation is more intelligible and it furnishes another good example of the simplification of mathematics by the use of an idealized arrangement.

Figure 42 represents a long portion (l centimeters long) of a very slender steel rod which is supposed to be so magnetized that a north pole of total strength m is uniformly spread over the portion. The lines of force of the magnetic field radiate cylindrically from such a pole, and it is easy to show from $\Phi = 4\pi m$ that the intensity of this cylindrically radiating field is $H = 2m/lr$ at a point distant r centimeters from the axis of the rod.

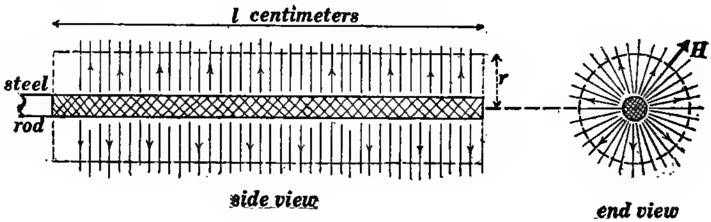


Fig. 42.

Imagine the linear "pole" above described to be placed axially in our long coil of wire. The cylindrically radiating magnetic field will engage, as it were, all of the turns in l centimeters of the coil (or Z turns), and, using r for the radius of the turns of wire, it is evident that a total length $2\pi rZ$ of wire will be pushed sidewise (end wise with respect to the coil) by the cylindrically radiating magnetic field. Therefore, using equation (6), considering that the force with which the linear pole acts on the wire is equal and opposite to the force with which the coil acts on the pole, and remembering that the force acting on the pole is equal to mH where H is the field due to the coil, we get equation (7).

PROBLEMS.

16. Figure 43a shows a magnet placed near a long straight wire carrying current away from the reader. Draw an arrow

showing force exerted by the wire on each pole of the magnet, and draw a diagram showing the resultant single force exerted on the magnet by the wire.

17. Figure 43*b* shows a magnet near a long straight wire

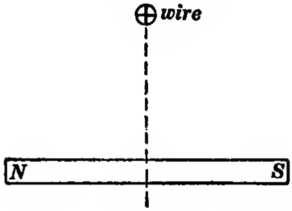


Fig. 43*a*.

Fig. 43*b*.

carrying current towards the reader. The distance of the *N*-pole from the wire is 10 centimeters and the length *NS* is 20 centimeters. The wire exerts a force of 900 dynes on the *N*-pole. Find value and direction of the force exerted by the wire on the *S*-pole. What is the value and direction of the force exerted on the wire by the magnet?

Note. The force exerted on either magnet pole by the long straight wire is equal and opposite to the force exerted on the wire by the pole, but the two forces do not have the same line of action! It would seem therefore that the principle of equality of action and reaction (which is not only that action and reaction are equal and opposite but that action and reaction have the same line of action) would not apply to Fig. 43*b*, *but if one considers the entire circuit of wire, and no less than an entire circuit ever exists magnetically, this difficulty vanishes.*

18. Specify the direction of the side push exerted on the wire

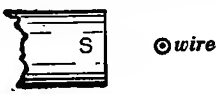


Fig. 44.

Fig. 45.

by the magnet pole in Fig. 44. Specify the direction of the side push on the wire in Fig. 45.

Note.—The direction of the field due to *S* in Fig. 44 is towards *S*; the field due to the wire circles round the wire in the direction in which a right-handed screw would have to be turned to travel in the direction of the current; and the wire is pushed away from the side on which these two fields are in the same direc-

tion. This rule may be understood by a careful study of Figs. 25*a*, 25*b* and 31. Most mechanically minded men prefer the right-handed screw convention to the rule which makes use of thumb and first two fingers of the right hand.

19. The armature of a direct-current dynamo has a length (under the pole faces) of 30 centimeters, and 250 of the armature wires are under the pole faces at all times. The radius of the armature (measured out to the wires) is 20 centimeters. The magnetic field in the air gaps between pole faces and armature core has an intensity of 6,000 gausses. Each armature wire carries a current of 75 amperes (see Fig. 37). Calculate the side force acting on each wire and calculate the total torque acting on the armature in dyne-centimeters.

20. A thin wooden block 25 × 40 cm. has 10 turns of wire wound around its edge, and the block is balanced in a horizontal position on an east-west axis with its 40 cm. edges north and south. A current of 28 amperes is sent through the winding (using very flexible leads). Find the distance from the balance axis at which a 9800-dyne weight would have to be hung to balance the torque action of the earth's magnetic field. Horizontal component of earth's field being 0.21 gauss, and vertical component 0.48 gauss.

21. Two straight parallel wires are 20 centimeters apart. One wire carries a current of 100 amperes and the other carries a current of 250 amperes (both currents in same direction). Do the wires attract or repel each other? How much force is exerted on 100 centimeters of either wire by the other?

22. Imagine an isolated pole of strength m to be placed at the center of a circle of wire in which a current of I abamperes of current is flowing, the radius of the circle being r. Write down an expression for the intensity at the wire of the magnetic field due to the pole, then derive an expression for the force exerted on the wire by the pole and specify the direction of this force, then get an expression for the force exerted on the pole by the wire, and then derive an expression for the intensity at the pole of the magnetic field due to the wire.

CHAPTER II.

CHEMICAL EFFECT OF THE ELECTRIC CURRENT.

24. **The chemical effect of the electric current again considered.**—A very beautiful experiment showing the deposition of a metal by the electric current is as follows: Two strips of lead are connected to seven or eight dry cells (in series) and dipped into a solution of lead nitrate,* as shown in Fig. 46. The flow of

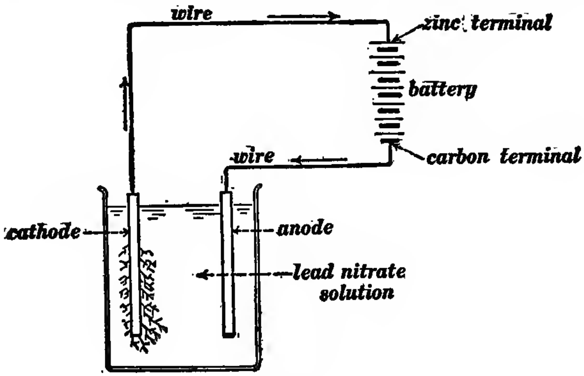


Fig. 46.

current decomposes the lead nitrate and deposits beautiful feather-like crystals of metallic lead on the lead strip which is connected to the zinc terminal of the battery.

This decomposition of a solution by an electric current is called *electrolysis*, and the solution which is decomposed is called an *electrolyte*. Electrolysis is usually carried out in a vessel provided with two plates of metal or carbon. Such an arrangement is called an *electrolytic cell*, and the plates of metal or carbon are called the *electrodes*. Thus the vessel which contains the lead nitrate solution and the two lead strips in Fig. 46 is an

* Ordinary sugar of lead (lead acetate), which can be obtained at any drug store, may be used instead of lead nitrate.

electrolytic cell. The electrode at which the current enters the solution is called the *anode,* and the electrode at which the current leaves the solution is called the *cathode.**

The chemical action which is produced by the electric current in the electrolytic cell of Fig. 46 is as follows: The lead nitrate ($PbNO_3$) is separated into two parts, namely, Pb (lead) and NO_3 (nitric acid radical). The lead (Pb) is deposited on the cathode, and the nitric acid radical (NO_3) is set free at the anode where it combines with the lead of the anode, forming a fresh supply of lead nitrate which is immediately dissolved in the electrolyte. That is to say, lead is deposited on the cathode and dissolved off the anode.

The chemical action produced by the electric current in an electrolytic cell takes place only in the immediate neighborhood of the electrodes.

The international standard ampere.—Very careful measurements have shown that one ampere deposits 0.001118 gram of silver per second from a solution of silver nitrate in water; and, inasmuch as it is very difficult to measure a current accurately in terms of its magnetic effect so as to get the value of the current directly in amperes, the ampere has been legally defined as the current which will deposit *exactly* 0.001118 gram of silver in one second from a solution of pure silver nitrate in water.

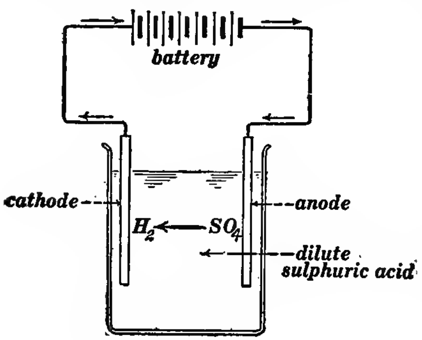


Fig. 47.

25. Another aspect of the chemical effect of the electric current. The voltaic cell or electric battery.—When electric

* The use of the terms *positive* and *negative* is extremely confusing because the *positive terminal* of a generator (a battery or dynamo) is the terminal out of which current flows, whereas the *positive terminal* of any receiver is understood to be the terminal at which current enters or flows into the receiver.

current flows through an electrolytic cell chemical action is produced. For example, Fig. 47 shows a battery forcing current through dilute sulphuric acid, the electrodes being plates of carbon or lead or platinum. The sulphuric acid (H_2SO_4) is decomposed by the current, being separated into H_2 (hydrogen) and SO_4 (sulphuric acid radical). The hydrogen appears at the cathode as bubbles of gas and escapes from the cell. The acid radical (SO_4) appears at the anode where it breaks up into SO_3 and O (oxygen). The oxygen appears in the form of bubbles and escapes from the cell, and the SO_3 combines with the water (H_2O) in the cell, forming H_2SO_4. The net result of the chemical action in the cell is therefore to decompose water (H_2O) inasmuch as hydrogen gas and oxygen gas are given off by the cell. Now by burning the hydrogen and oxygen heat energy can be obtained, and therefore it is evident that **work must be done** (*by the battery in Fig. 47*) **to decompose the H_2O in the electrolytic cell.**

Usually the chemical action which is produced by the current in an electrolytic cell requires the *doing of work* as above explained, that is to say, an electric generator (battery or dynamo) must be used to force the electric current through the electrolytic cell. **In some cases, however, the chemical action which is produced by the flow of current through the electrolytic cell is A SOURCE OF ENERGY.** In such a case it is not necessary to use a separate electric generator (battery or dynamo) to force electric current through the electrolytic cell, for such an electrolytic cell can maintain its own current through the electrolyte from electrode to electrode and through an outside circuit of wire which connects the electrodes. Such an electrolytic cell is called a *voltaic cell* or an *electric battery.* That is to say, **a voltaic cell is an electrolytic cell in which the chemical action produced by the flow of current is a source of energy.**

26. The simple voltaic cell.—The simplest example of an electrolytic cell in which the chemical action produced by the current is a source of energy, is the so-called *simple voltaic cell*

which is shown in Fig. 48. It consists of a carbon or copper electrode *C* and a clean zinc electrode *Z* in dilute sulphuric acid. The flow of current through this cell breaks up the H_2SO_4 into two parts, namely, H_2 (hydrogen) and SO_4 (sulphuric acid radical). The hydrogen appears at the carbon electrode and escapes as a gas, and the SO_4 appears at the zinc electrode where it combines with the zinc to form zinc sulphate ($ZnSO_4$). The combination of the zinc and the SO_4 supplies more energy than is required to separate the H_2 and SO_4, that is to say, the chemical action which is produced by the flow of current through the cell is a source of energy, and the cell itself maintains a flow of current.

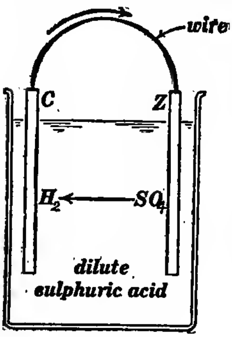


Fig. 48.

27. Voltaic action and local action.—Two distinct kinds of chemical action take place in a voltaic cell, namely, (*a*) the chemical action which depends upon the flow of current and which does not exist when there is no flow of current, and (*b*) the chemical action which is independent of the flow of current and which takes place whether the current is flowing or not.

The chemical action which depends on the flow of current is proportional to the current, that is to say, this chemical action takes place twice as fast if the current which is delivered by the voltaic cell is doubled. This chemical action is essential to the operation of the voltaic cell as a generator of current, its energy is available* for the maintenance of the current which is produced by the cell, and it is called *voltaic action.*

The chemical action which is independent of the flow of current does not help in any way to maintain the current; it represents a waste of materials, and it is called *local action.* Local action takes place more or less in every type of voltaic cell. It may be greatly reduced in amount, however, by using pure zinc, and especially by coating the zinc with a thin layer of metallic mercury (amalgamation).

* In general not wholly available.

Example of local action.—The zinc plate in the simple voltaic cell which is shown in Fig. 48 dissolves in the sulphuric acid even when no current is flowing through the cell, zinc sulphate and hydrogen are formed, and all of the energy of this reaction goes to heat the cell. If the zinc is very pure and if its surface is clean this chemical action takes place very slowly, but if the zinc is impure the action is usually very rapid. *The hydrogen which is liberated during this local action appears at the zinc plate.*

Example of voltaic action.—When the circuit in Fig. 48 is closed, *hydrogen bubbles begin to come off the carbon electrode*, and zinc sulphate is formed at the zinc electrode. This is voltaic action, and it ceases when the circuit is broken.

The essential and important feature of voltaic action is that it is reversed if a current from an outside source is forced backwards through the voltaic cell, provided no material which has played a part in the previous voltaic action has been allowed to escape from the cell. Thus in the simple voltaic cell, which is described in Art. 26, the sulphuric acid (H_2SO_4) is decomposed, zinc sulphate ($ZnSO_4$) is formed at the zinc electrode, and hydrogen is liberated at the carbon electrode. If a reversed current is forced through this simple cell, the zinc sulphate previously formed will be decomposed, metallic zinc will be deposited upon the zinc plate, and the sulphuric acid radical (SO_4) will be liberated at the carbon plate, where it will combine with the trace of hydrogen which is clinging to the carbon plate and form sulphuric acid (H_2SO_4). In this simple cell, however, the greater part of the liberated hydrogen has, of course, escaped, and the reversed chemical action due to a reversed current cannot long continue.

Local action, being independent of the current, is not affected by a reversal of the current.

28. Primary and secondary chemical reactions in the electrolytic cell.—The decomposition of the electrolyte is the direct or immediate or primary effect of the flow of current, therefore, the decomposition of the electrolyte may be spoken of as the *primary chemical action* in a electrolytic cell.

When the decomposed parts of the electrolyte appear at the electrodes, chemical action usually takes place between these parts and the electrodes or between these parts and the solution, and these chemical actions are called the *secondary chemical reactions* in the electrolytic cell. For example, the primary chemical reaction in the simple voltaic cell which is shown in Fig. 48 is the decomposition of the sulphuric acid into hydrogen (H_2) and sulphuric acid radical (SO_4); and the combination of the acid radical (SO_4) with the zinc of the electrode is a secondary reaction.

The primary chemical action in an electrolytic cell usually represents the doing of work **ON** the cell, and the secondary chemical reactions in an electrolytic cell usually represent the doing of work **BY** the cell; therefore secondary reactions are very important in the voltaic cell or electric battery.

29. The use of oxidizing agents in the voltaic cell.—The combination of the SO_4 with the zinc of the anode is the secondary chemical action in the simple voltaic cell which is shown in Fig. 48. The available energy of the total chemical action which takes place in this cell may be greatly increased, however, by providing an oxidizing agent in the neighborhood of the carbon electrode so that the hydrogen may be oxidized and form water (H_2O) at the moment of its liberation by the current. The energy of this oxidation increases the available energy of the chemical action as a whole, and greatly strengthens the cell as a generator of electric current.

30. The chromic acid cell.—The *Grenet cell* is similar to the simple voltaic cell, as shown in Fig. 48, except that the electrode *C* is of carbon, and chromic acid is added to the electrolyte to furnish oxygen for the oxidation of the hydrogen as it is set free at the carbon electrode. There is, however, a very rapid waste of zinc in this cell by local action even when the zinc is amalgamated, and the cell is now seldom used. A modified form of the Grenet cell, known as the *Fuller cell*, is shown in Fig. 49. In this cell the electrolyte *e* is dilute sulphuric acid, the zinc anode

Z is contained in a porous earthenware cup, and the chromic acid is dissolved only in that portion of the electrolyte which surrounds the carbon cathode *C*. In this cell there is not a rapid waste of zinc by local action.

31. Open-circuit cells and closed-circuit cells. — A voltaic cell which can be left standing unused, but in readiness at any time for the delivery of current when its circuit is closed, is called an *open-circuit cell.* A cell to be suitable for use as an open-circuit cell should above all things be nearly free from local action. The cell most extensively used for open-circuit service is the ordinary dry cell.

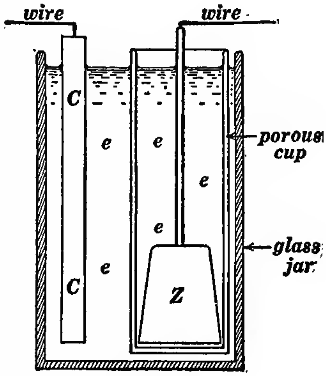


Fig. 49.
The Fuller cell.

A voltaic cell which is suitable for delivering a current steadily is called a *closed-circuit cell.* The gravity Daniell cell which is described in Art. 33 was formerly much used as a closed-circuit cell, but the storage battery is now displacing all other types of voltaic cells for the delivery of fairly large or steady current.

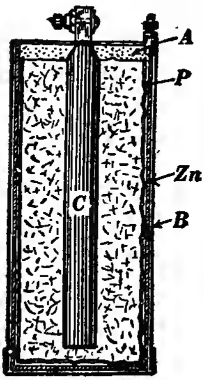


Fig. 50.
The dry cell.

32. The ordinary dry cell. — A sectional view of this cell is shown in Fig. 50. The containing vessel is a can made of sheet zinc. This can serves as one electrode of the cell, and a binding post is soldered to it. The zinc can is lined with several thicknesses of blotting paper *P*, and the space between the blotting paper and the carbon rod *C* is packed with bits of coke and manganese dioxide. The porous contents of the cell are then saturated with a solution of am-

monium chloride (sal ammoniac), and the cell is sealed with asphaltum cement *A*. The zinc can is usually protected by a covering of pasteboard *B*. The dry cell has been humorously defined as a voltaic cell which, being hermetically sealed, is always wet; whereas the old-style wet cell was open to the air and frequently became dry.

Reputable manufacturers always stamp the date of manufacture on their dry cells, and a purchaser should not accept a cell which is much more than one or two months old. The condition of a dry cell is most satisfactorily indicated by observing the current delivered when the cell is momentarily short circuited* through an ammeter. When the cell has been exhausted by use or when it has dried out by being kept too long, the short-circuit current is greatly reduced in value. An ordinary dry cell, when fresh, should give about 25 or 30 amperes on a momentary short circuit when the cell is at ordinary room temperature.

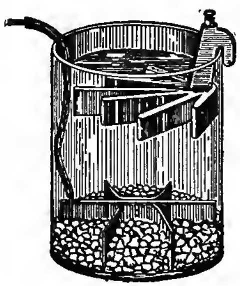


Fig. 51.
The gravity cell.

33. The gravity Daniell cell.—This cell consists of a copper cathode at the bottom of a glass jar and a zinc anode at the top as shown in Fig. 51. The electrolyte is mainly a solution of zinc sulphate. Crystals of copper sulphate are dropped to the bottom of the cell and a dense solution of copper sulphate surrounds the copper cathode.

This cell has a considerable amount of local action when it is allowed to stand unused, because of the upward diffusion of the copper sulphate. The cell is used in telegraphy and for operating the "track circuit" relays in automatic railway signalling.

PROBLEMS.

23. The anode of an electrolytic cell is a copper rod one inch in diameter, and the cathode is a hollow copper cylinder 6 inches

* That is to say, the very low resistance ammeter is connected directly to the terminals of the cell.

in diameter. The two electrodes are co-axial and they stand on a flat glass plate in an electrolyte which is 8 inches deep over the glass plate. A current of 5 amperes flows through the cell. Find the current density at the cathode and the current density at the anode.

Note.—The current density on an electrode is the current per unit of area, and it is generally non-uniform. In the conditions specified in the problem, however, the current density is uniform over the surface of each electrode. Current density at an electrode is important inasmuch as the character of the chemical action at an electrode depends in many cases on current density, and the physical character of a deposited metal depends on the current density.

24. How long a time in seconds would be required for one ampere to deposit one gram-equivalent (107.93 grams) of silver from a solution of pure silver nitrate?

25. A current of I amperes will deposit $0.000338 \times I$ grams of zinc per second from a solution of zinc sulphate, or cause the consumption of $0.000338 \times I$ grams of zinc per second in a gravity Daniell cell. The chemical action which takes place in the gravity Daniell cell is identical to what takes place as "local action" when zinc filings are stirred winto a solution of copper sulphate, and experiment shows that 753 calories or 3,160 joules of heat is developed when one gram of zinc filings is stirred into a copper sulphate solution. Assuming that all of the energy of the chemical action in the Daniell cell is available for maintaining current, find joules-per-second-per-ampere available for maintaining current.

Note.—Joules-per-second-per-ampere or watts-per-ampere is, of course, a quantity which gives watts when multiplied by amperes, and joules-per-second-per-ampere is called *electromotive force*. The calculated answer to this problem is the electromotive force of the gravity Daniell cell in volts. One volt is one joule-per-second-per-ampere.

26. The electromotive force of the chromic acid cell is about 2.25 volts, that is to say, the cell does 2.25 joules of work per second per ampere of delivered current. Assuming that all of the energy of the chemical action in the cell (voltaic action) is represented by the work the cell does in maintaining current, calculate the rise of temperature produced when one gram of

zinc filings is stirred into 500 grams of dilute sulphuric and chromic acids, mixed. Take the specific heat capacity of this dilute acid to be the same as water.

Note.—Zinc consumed by voltaic action in any form of voltaic cell is 0.000338 gram per second per ampere.

27. A Grenet form of chromic acid cell delivers an average current of 5 amperes for $2\frac{1}{2}$ hours. At the beginning the mass of the zinc plate was 150 grams and at the end of the run the mass of the zinc plate was 75 grams. What percentage of the total zinc consumption is due to voltaic action and what percentage is due to local action?

---

34. **The storage cell.**—A voltaic cell may be completely repaired after use by forcing a current backwards through the cell, if there is no local action in the cell, and if all of the materials which take part in the voltaic action remain in the cell. A voltaic cell which meets these two conditions is called a *storage cell.* The process of repairing the cell by forcing a current through it backwards is called *charging*, and the use of the cell for the delivery of current is called *discharging.*

The lead storage cell.—The voltaic cell which is most extensively used as a storage cell is one in which one electrode is lead peroxide (PbO_2), the other electrode is spongy metallic lead (Pb), and the electrolyte is dilute sulphuric acid (H_2SO_4). This cell is called the *lead storage cell.* The lead peroxide and the spongy metallic lead are called the *active materials* of the cell. These active materials are porous and brittle, and they are usually supported in small grooves or pockets in heavy plates or grids of metallic lead. These lead grids serve not only as mechanical supports for the active materials, but they serve also to deliver current to or receive current from the active materials which constitute the real electrodes of the cell.

As a lead storage cell is discharged, the active material on both electrodes is reduced to lead sulphate $PbSO_4$; and when the cell

is charged, the lead sulphate on one grid is converted back into spongy metallic lead, and the lead sulphate on the other grid is converted back into lead peroxide.*

35. **Definition of electrochemical equivalent. Chemical calculations in electrolysis.**—The amount of silver deposited per second in the operation of silver plating is proportional to the strength of the current in amperes, and the amount of silver deposited in one second by one ampere is called the *electrochemical equivalent* of silver; it is equal to 0.001118 gram per ampere per second, or 4.025 grams per ampere per hour.

In the great majority of cases no material is actually deposited at either electrode in the electrolytic cell, but chemical action is always produced in the immediate neighborhood of the electrodes, and the amount of chemical action which takes place in a given time due to the flow of a given current through the cell can be calculated from very simple data. A statement of the method employed in this calculation involves a number of chemical terms, and these terms are exhibited in the following schedules.

The valencies of various chemical elements, acid radicals and so forth are shown by the numbers in the following exhibit. Thus one atom of hydrogen combines with one atom of chlorine and each has a valency of 1, whereas one atom of copper combines with two atoms of chlorine in the formation of cupric chloride so that the valency of cupric copper is 2. The valency of cuprous copper, however, is 1. The valency of the sulphuric acid radical (SO_4) is 2. No attempt is made here to give a general definition of valency but merely to recall to the student's mind the knowledge of valency which he has obtained from his study of chemistry.

* A further discussion of batteries and their uses is given in W. S. Franklin's *Elements of Electrical Engineering*, Vol. II, Franklin and Charles, Bethlehem, Pa., 1912. The Edison nickel-iron storage cell is discussed on pages 205–209; a discussion of battery costs is given on pages 208–211; directions for the management and care of the lead storage battery are given on pages 211–214; and the uses of the storage battery are discussed on pages 214–255.

Exhibit of Valencies.

Name	Hydrochloric acid		Sodium chloride		Cupric chloride		Cuprous chloride	
Chemical symbol Valency	H 1	Cl 1	Na 1	Cl 1	Cu 2	Cl_2 2×1	Cu 1	Cl 1
Name	Sulphuric acid		Sodium sulphate		Cupric sulphate		Cuprous sulphate	
Chemical symbol Valency	H_2 2×1	SO_4 2	Na_2 2×1	SO_4 2	Cu 2	SO_4 2	Cu_2 2×1	SO_4 2

Let m be the atomic weight of an element, or the molecular weight of an atomic aggregate or group such as the acid radical SO_4 or such as the base radical NH_4 (which occurs in ammonium chloride, NH_4Cl), and let v be the valency of the element or aggregate. *Then* m/v *grams of the element or aggregate is called a* **chemical equivalent** *thereof*. The chemical equivalents of a few elements and aggregates are shown in the following exhibit.

Exhibit of chemical equivalents in grams.

Symbol of substance...	H	Na	Ag	Cl	NO_3	SO_4	Cu*	Cu†	Zn	Al
Atomic or molecular weight..............	1	23	108	35.5	62	96	63.6	63.6	65.4	27.1
Valency..............	1	1	1	1	1	2	2	1	2	3
Chemical equivalent in grams..............	1	23	108	35.5	62	48	31.8	63.6	32.7	9.03

* Cupric copper, that is copper as it exists in ordinary cupric sulphate, $CuSO_4$.

† Cuprous copper.

Note.—Atomic weights are given only approximately in round numbers for the sake of simplicity.

I. **The amount of chemical action which takes place in an electrolytic cell is proportional to the current and to the time that the current continues to flow,** that is to say, the amount of chemical action is proportional to the product of the current and the time. This product may be expressed in *ampere-seconds* or in *ampere-hours*. Thus ten amperes flowing for five hours constitutes what is called 50 ampere-hours.

II. *To deposit one chemical equivalent of silver, that is* 108 *grams of silver, requires* 26.82 *ampere-hours, and* **26.82 ampere-hours will liberate one chemical equivalent of any element or radical at an electrode in an electrolytic cell.** For example:

26.82 AMPERE-HOURS WILL LIBERATE

at the anode	at the cathode
62 grams of NO_3 from nitric acid or any nitrate solution.	23 grams of Na from a solution of NaOH, or from a solution of any sodium salt.
48 grams of SO_4 from sulphuric acid or any sulphate solution.	31.8 grams of Cu from a solution of any cupric salt.
35.5 grams of Cl from hydrochloric acid or any chloride solution.	63.6 grams of Cu from a solution of any cuprous salt.
17.01 grams of OH from a solution of caustic soda or potash.	9.03 grams of Al from a solution of any aluminum salt.
etc.	etc.

EXAMPLE OF ELECTROCHEMICAL CALCULATIONS.

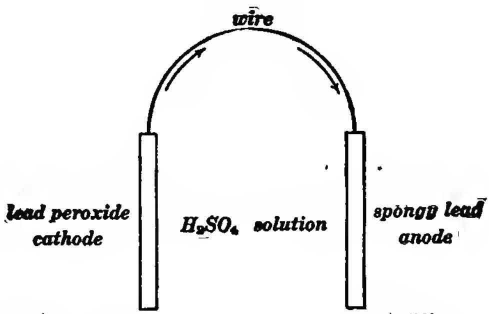
Note.—When one of the elements or radicals which take part in a chemical reaction is *di-valent* the electro-chemical calculations are simplified by considering the amount of chemical action which is produced by *two* times 26.82 ampere-hours. When one of the elements or radicals is *trivalent* the calculations are simplified by considering the amount of chemical action which is produced by *three* times 26.82 ampere-hours. Thus in the following example di-valent elements and radicals are involved, and the chemical action produced by 53.64 ampere-hours is taken as the basis of the calculations; 53.64 ampere-hours liberates *two chemical equivalents* of material at each electrode.

Example.—One result of the discharge of a lead storage cell is that H_2SO_4 from the solution combines with the active material of the electrodes thus reducing the strength of the solution. Let it be required to find how much H_2SO_4 is taken from the solution by 53.64 ampere-hours of discharge. The sketch and schedule on page 48 shows the reactions at both electrodes. From this schedule we see that $(96 + 2)$ grams of H_2SO_4 is taken from the solution by the electrolytic action, and that another 98 grams of H_2SO_4 is taken from the solution by the reaction at the cathode. Therefore a total of 196 grams of H_2SO_4 is taken from the solution. Furthermore 36 grams of H_2O is produced by the reaction at the cathode. Therefore the solution is weakened by the taking of H_2SO_4 from it and by the adding of H_2O to it.

PROBLEMS.

28. Find the current density required to deposit a layer of metallic copper 0.1 millimeter thick on a flat metal plate from a solution of cupric sulphate in 3 hours, density of copper being 8.9 grams per cubic centimeter.

29. Find the time required for 10 amperes to liberate two



$$\overbrace{\underset{36}{2H_2O}+\underset{303}{PbSO_4}=\underset{98}{H_2SO_4}+\underset{239}{PbO_2}+\underset{2}{2H}}^{\text{grams}}\xleftarrow[\text{ampere-hours}]{63.64}\overbrace{\underset{96}{SO_4}+\underset{207}{Pb}=\underset{303}{PbSO_4}}^{\text{grams}}$$

cubic feet (5.074 grams) of hydrogen and one cubic foot of oxygen in an electrolytic cell like Fig. 47.

30. Consider the hydrogen and oxygen which is liberated by 10 amperes in 26.8 hours in an electrolytic cell like Fig. 47, namely, 1 gram of hydrogen and 8 grams of oxygen. If we should burn this hydrogen and oxygen we would get 34,700 calories or about 146,000 joules of heat. Assuming that the work done in forcing the 10 amperes through the electrolytic cell is *all* represented by this heat of combustion, calculate the joules-per-second-per-ampere required to force the 10 amperes through the electrolytic cell.

Note.—The joules-per-second-per-ampere as here calculated is the electromotive force in volts that would be required to push current through the electrolytic cell shown in Fig. 47 under the assumption that the work done is equal to the heat-energy that may be obtained by re-combining the hydrogen and oxygen. As a matter of fact an electromotive force of about 2.0 or 2.1 joules-per-second-per-ampere (volts) is required.

31. The cost of gravity-cell zinc is, say, 6 cents per pound, and the cost of copper sulphate crystals ($CuSO_4 + 5H_2O$) is, say, 6.5 cents per pound. Half of the materials consumed in a gravity cell is wasted by local action and about one third of the zinc is left as scrap and is worth about 2 cents per pound. Furthermore the copper which is deposited on the cathode is worth, as scrap, about 10 cents per pound. The terminal electromotive force of the cell while it is delivering 0.16 ampere is about 0.72 volt.* What is the cost per kilowatt-hour of the output of the cell making no allowance for labor required to take care of the cell and making no allowance for interest on cost of glass jar. Ans. $1.73.

32. A lead storage cell delivers 10 amperes for 8 hours. Find increase in weight of each electrode.

* About 0.72 joule per ampere per second.

CHAPTER III.

THE HEATING EFFECT OF THE ELECTRIC CURRENT.

36. Heating effect of the electric current again considered. Joule's law.—The heating effect of the electric current is mentioned in Art. 1. The lamp filament in Fig. 2 is heated to a high temperature by the current. Careful observation shows that every portion of an electric circuit is heated move or less by the electric current. Thus the connecting wires in Fig. 2 are heated to some extent.

It is important to understand that the heating effect of the electric current is *the generation of heat in each portion of a circuit at a definite rate*, so many calories or joules per second; and each portion of an electric circuit grows hotter and hotter until it gives off heat to its surroundings as fast as heat is generated in it by the current.

A portion of an electric circuit is said to have a *high resistance* if a large amount of heat is generated in it per second by a given current, or a *low resistance* if a small amount of heat is generated in it per second by a given current. Thus a great deal more heat is generated per second in the lamp filament in Fig. 2 than in the connecting wires so that the resistance of the lamp filament is much greater than the resistance of the connecting wires.

Joule's law.—An important discovery was made by James Prescott Joule about 1850 concerning the relation between the strength of a current in amperes and the rate at which heat is generated by the current. Joule found that *the rate of generation of heat in a given piece of wire is proportional to the square of the current flowing through the wire* (Joule's law).

To say that the rate of generation of heat in a given wire (or other conductor) is *proportional* to the square of the current is the same thing as to say that the rate of generation of heat is

equal to the square of the current multiplied by a constant factor, *a factor which has a certain definite value for the given piece of wire.* Let us represent this factor (for a given piece of wire) by the letter R. Then RI^2 is the rate of generation of heat in the wire by a current of I amperes, that is RI^2 is the amount of heat generated in the wire per second, and RI^2t is the total amount of heat generated in the wire during t seconds. Therefore we may write:

$$H = RI^2t \tag{8}$$

where H is the amount of heat generated in a wire during t seconds by a current of I amperes, and R is a factor which has a definite value for the given piece of wire.

Example.—The amount of heat developed in a certain electric lamp in 10 minutes, as found by a calorimeter, is 7143 calories or 30,000 joules, when 0.51 ampere flows through the lamp. Therefore, putting H = 30,000 joules, I = 0.51 ampere, and t = 600 seconds in equation (8), we get R equal to 192.2 joules-per-(ampere)^2-per-second; but *one-joule-per-(ampere)^2-per-second* is called an *ohm.* Therefore the given electric lamp has a resistance of 192.2 ohm.

Definition of the ohm.—*A wire is said to have a resistance of one ohm when one joule of heat is generated in it per second by a current of one ampere.*

Definition of the abohm.—*A wire is said to have a resistance of one abohm when one erg of heat is generated in it per second by a current of one abampere.* One ohm is equal to 10^9 abohms.

37. Dependence of resistance upon the length and size of a wire. Definition of resistivity.—The resistance R of a wire of given material is directly proportional to the length l of the wire, and inversely proportional to the sectional area s of the wire; that is

$$R = k\frac{l}{s} \tag{9}$$

in which k is a constant for a given material, and it is called

the *resistivity** of the material. The exact meaning of the factor k may be made apparent by considering a wire of unit length ($l = 1$) and of unit sectional area ($a = 1$). In this case R is numerically equal to k, that is to say, *the resistivity of a material is numerically equal to the resistance of a wire of that material of unit length and unit sectional area.*

Electrical engineers nearly always express lengths of wires in feet and sectional areas in circular mils.† If equation (9) is used

TABLE.—Resistivities and Temperature Coefficients.

	a	b	β
Aluminum wire (annealed) at 20° C.	27.4×10^{-7}	16.5	+0.0039
Copper wire (annealed) at 20° C.	17.24×10^{-7}	10.4	+0.0040
Iron wire (pure annealed) at 20° C.	95×10^{-7}	58.0	+0.0045
Steel telegraph wire at 20° C.	150×10^{-7}	91†	+0.0043†
Steel rails at 20° C.	120×10^{-7}	72†	+0.0035†
Mercury at 0° C.	943.4×10^{-7}	—	+0.00088
Platinum wire at 0° C.	89.8×10^{-7}	54.0	+0.00354
German-silver wire at 10° C.	212×10^{-7}	127†	+0.00025†
Manganin wire (Cu 84, Ni 12, Mn 4) at 20° C.	475×10^{-7}	286	
"Ia Ia" metal wire, hard (copper-nickel alloy) at 20° C.	500×10^{-7}	300†	−0.00001†
"Climax" or "Superior" metal (nickel-steel alloy) at 20° C.	800×10^{-7}	480†	+0.00067†
Arc-lamp carbon at ordinary room temperature	0.005		−0.0003†
Sulphuric acid, 5 per cent. solution at 18° C.	4.8 ohms		−0.0120*
Ordinary glass at 0° C. (density 2.54)	10^{15} ohms†		
Ordinary glass at 60° C.	10^{12} ohms†		
Ordinary glass at 200° C.	10^{8} ohms†		

a = resistance in ohms of a bar 1 centimeter long and 1 square centimeter sectional area.

b = resistance in ohms of a wire 1 foot long and 0.001 inch in diameter.

β = temperature coefficient of resistance per degree centigrade (mean value between 0° C. and 100° C.).

Near ordinary room temperature the resistance of an maganinn wire is very nearly independent of temperature.

* Between 18° C. and 19° C.

† These values differ greatly with different samples.

to calculate the resistance of a wire in ohms when the length l of the wire is expressed in feet and the sectional area s in circular

* Sometimes called *specific resistance.* The reciprocal of the resistivity of a substance is called its *conductivity.*

† One *mil* is a thousandth of an inch. One *circular mil* is the area of a circle of which the diameter is one mil. The area of any circle in circular mils is equal to the square of the diameter in mils. Thus a wire 100 mils in diameter has a sectional area of 10,000 circular mils.

mils, then the value of k must be the resistance of a wire of the given material one foot long and one circular mil in sectional area. The accompanying table gives the resistivities of the more important substances together with their temperature coefficients of resistance.

PROBLEMS.

33. An electric heater of the immersion type is placed in a vessel containing 5,000 grams of water, and the temperature of the water is raised from 0°C. to 40° C. in five minutes with a current of 25.5 amperes flowing through the heater. What is the resistance of the heater in ohms? All the work done in pushing the current through the heater is represented by the heat that is developed; how many joules-per-second-per-ampere are used in pushing the current through the heater?

Note.—One joule-per-second-per-ampere is called a volt.

34. The field coil of a dynamo contains 11,340 grams of copper (specific heat 0.094), weight of cotton insulation negligible. The resistance of the coil is 100 ohms. At what rate does the temperature of the coil begin to rise when a current of 0.5 ampere is started in the coil?

Note.—At the start, when the coil is at the same temperature as the surrounding air, all of the heat generated in the coil is used to raise the temperature of the coil; later, when the coil has become warmer than the air, a portion of the heat generated in the coil is given off to the air. The problem is simply this: How fast would all the heat generated in the coil cause the temperature of the coil to rise?

35. A given piece of copper wire has a resistance of 5 ohms, another piece of copper wire is 1.5 times as long but it has the same weight (and volume) as the first piece. What is its resistance?

36. A given spool wound full of copper wire 60 mils in diameter has a resistance of 3.2 ohms. An exactly similar spool is wound full of copper wire 120 mils in diameter; what is its resistance?

Note.—The spool will contain half as many layers and half as many turns in each layer of the larger wire, and the mean length of one turn of wire is the same in each case.

37. What is the resistance at 20° C. of 2 miles of commercial copper wire 300 mils in diameter?

38. Find the resistance at 20° C. of a copper conductor 100 feet long having a rectangular section 0.5 inch by 0.25 inch.

Note.—The area of a circle d mils in diameter is d^2 circular mils, or $\frac{\pi}{4}\left(\frac{d}{1000}\right)^2$ square inches. Therefore the sectional area of a rectangular bar in square inches must be multiplied by $\frac{4,000,000}{\pi}$ to reduce to circular mils.

39. What is the resistance at 20° C. of a wrought iron pipe 20 feet long having one inch inside diameter and 1.25 inches outside diameter.

Note.—Use resistivity of pure annealed iron.

40. Calculate the resistance in ohms of an arc lamp carbon 0.5 inch in diameter and 12 inches long.

41. Calculate the resistance of a column of 5 per cent solution of sulphuric acid at 18° C., the length of the column being 20 centimeters and the sectional area being 12 square centimeters.

42. Current enters a thin circular disk of platinum at the center of the disk and flows radially outwards towards the edge of the disk. The thickness of the disk is h centimeters, its radius is r centimeters, and its resistance is R ohms. Find an expression for $\frac{dR}{dr}$.

38. **Variation of resistance with temperature.**—The electrical resistance of a conductor which forms a portion of an electrical circuit varies with temperature. Consider, for example, (*a*) an iron wire, (*b*) a copper wire, (*c*) a platinum wire, (*d*) a German-silver wire, (*e*) a carbon rod, and (*f*) a column of dilute sulphuric acid, *each of which has a resistance of 100 ohms at 0° C.* Then the values of the resistances of (*a*), (*b*), (*c*), (*d*), (*e*) and (*f*) at other temperatures, as determined by experiment, are shown by the ordinates of the curves in Fig. 52. As shown in this figure, iron and copper increase greatly in resistance with rise of tem-

perature, and German silver increases only slightly in resistance with rise of temperature. But carbon and sulphuric acid decrease in resistance with rise of temperature, carbon to a very slight extent and dilute sulphuric acid to a very great extent.

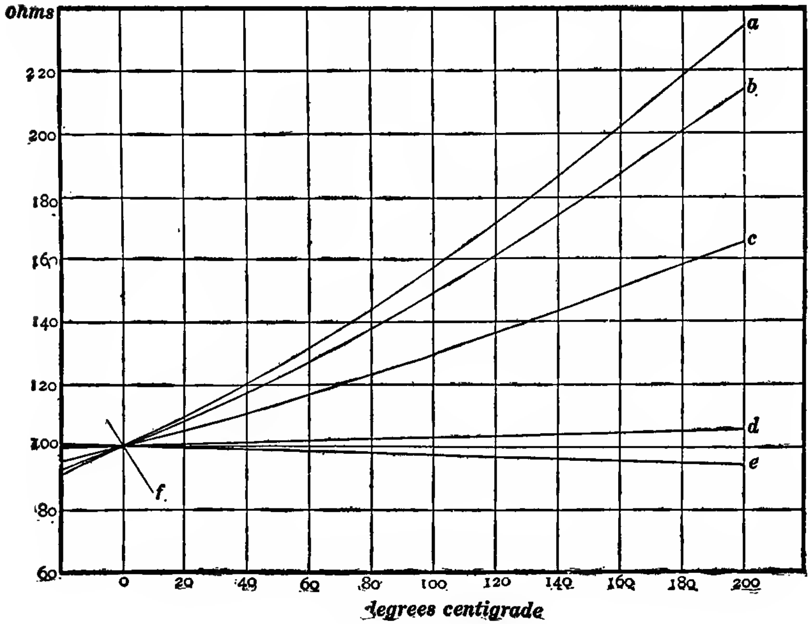


Fig. 52.

For most practical purposes the curves in Fig. 52 may be thought of as straight lines so that:

$$R_t = R_0(1 + \beta t) \tag{10}$$

where R_0 is the resistance of a wire at 0° C., R_t is the resistance of the wire at t° C. and β is a constant for the material of which the wire is made. This constant β is called the *temperature coefficient of resistance* of the material. The values of β for various materials are given in the table in Art. 37.

PROBLEMS.

43. A coil of copper wire has a resistance of 5 ohms at 20° C. What is its resistance at 0° C., and what is its resistance at 90° C.?

44. A wire has a resistance of 164.8 ohms at 0° C. and a resistance of 215.2 ohms at 85° C. What is the mean temperature coefficient between 0° C. and 85° C.?

45. The field coil of a dynamo has a resistance of 42.6 ohms after the dynamo has stood for a long time in a room at 20° C. After running for several hours the resistance of the coil is 51.6 ohms. What is its temperature?

46. A carbon-filament glow lamp has a resistance of 277 ohms at 0° C., and a resistance of 220 ohms at 1,000 °C. What is the mean temperature coefficient of resistance of the filament between 0° C. and 1,000° C.?

47. The curves in Fig. 52 can be expressed more accurately by an equation of the form $R_t = R_0(1 + at + bt^2)$ than by the simpler equation $R_t = R_0(1 + \beta t)$. A sample of very pure annealed platinum wire has a resistance of 124.3 ohms at 0° C., 242.38 ohms at 250° C., and 338.8 ohms at 500° C. Find the values of the coefficients a and b.

48. A coil of very pure annealed platinum wire has a resistance of 24.62 ohms at 0° C., and when the wire is placed in a furnance and protected from the furnace gases by a porcelain tube it has a resistance of 120.8 ohms. Find the temperature of the furnace using the equation $R_t = R_0(1 + at + bt^2)$, where $a = +0.00394$ and $b = -0.000,000,584$.

39. Power required to maintain a current in a circuit IN WHICH ALL OF THE ENERGY REAPPEARS IN THE CIRCUIT IN THE FORM OF HEAT IN ACCORDANCE WITH JOULE'S LAW.—Work must of course be done in forcing an electric current through an electric motor, but all of the work so done does not reappear in the motor wires as heat; a large portion reappears at the motor pulley and is delivered as mechanical energy to the machine which is driven by the motor.

Work also must be done in forcing an electric current backwards through an exhausted storage battery (to charge the battery), but all of the work so done does not reappear as heat in the circuit, a large portion of the work is expended in bringing about the chemical action which takes place as the battery is charged.

When a current is maintained in a simple circuit of wire, or in

a circuit containing glow lamps, all of the work done in maintaining the current *does* reappear in the circuit as heat, *and the rate at which work is done in maintaining the current is equal to the rate at which heat energy appears in the wire.* Now heat energy is generated by a current of I amperes at the rate of RI^2 joules per second in a circuit of which the resistance is R ohms. Therefore, to maintain a current of I amperes in a circuit having a resistance of R ohms work must be done at the rate of RI^2 joules per second. That is:

$$P = RI^2 \tag{11}$$

where P is the power in watts (or joules per second) required to maintain a current of I amperes in a circuit of which the resistance is R ohms. **Equation (11) is true only when all of the work expended in maintaining the current reappears in the circuit as heat in accordance with Joule's law.**

Example.—A certain electric glow lamp has a resistance when hot* of 192.2 ohms. To calculate the power required to maintain a current of 0.51 ampere through the lamp, we multiply 192.2 ohms by (0.51 ampere)² which gives 50 watts.

40. Electromotive force.—We think of an electric generator (battery or dynamo) as a kind of pump forcing a current of electricity through a circuit, the flow of current being opposed by a kind of resistance; that is to say, we think of a battery or dynamo as exerting a kind of propelling force on the current. This propelling force is called *electromotive force,* **and it is expressed numerically as so many joules of work done per second per ampere of current flowing.** If E is the rate at which a battery does work *per ampere of current,* then the total rate at which the battery does work must be EI joules per second or EI watts. That is

$$P = EI \tag{12}$$

where P is the rate at which a battery does work, E is the

* The resistance of a glow lamp changes greatly with the temperature of the filament.

electromotive force of the battery (expressed in volts as we shall see) and I is the current flowing through the battery.

The electromotive force of any generator (battery or dynamo) is the factor E by which the current I must be multiplied to give the power output P of the generator.

Remark.—It may seem from equation (11) that the power P delivered to a circuit should be proportional to the square of the current; but to increase the current delivered by a given battery the resistance of the circuit must be decreased, that is to say, *as relating to a given battery* R in equation (11) is not constant if I varies.

41. **Ohm's law.**—The rate at which energy is delivered by a battery is EI watts, as above explained, and the rate at which heat is prduced in the circuit is RI^2 watts according to Art. 36. Therefore, *if all the energy supplied by the battery is converted into heat in the circuit in accordance with Joule's law*, then the power developed by the battery must be equal to the rate at which heat is generated in the circuit, that is, EI must be equal to RI^2, or cancelling I, we must have:

$$E = RI \tag{13a}$$

or, solving for I, we have:

$$I = \frac{E}{R} \tag{13b}$$

These two relations were discovered by G. S. Ohm in 1827, and they constitute what is known as *Ohm's law.*

Ohm's law is true when all of the energy delivered by an electric generator is used to heat the circuit, that is when $EI = RI^2$. Ohm's law is not true when a portion of the energy delivered by the generator is used to drive a motor or to produce chemical action as in the charging of a storage battery.

Remark.—Electrical engineers frequently speak of the equation which expresses the relation between voltage and current in an alternating-current circuit as a generalized form of Ohm's

law because the equation which expresses this relation is similar in form to equation (13).

The definition of electromotive force which is given in Art. 40 and which is involved in equation (12) is entirely rigorous and correct, and yet it seems somewhat vague. Therefore it may be helpful to consider the familiar gravity Daniell cell or battery which is described in Art. 33. Let z be number of grams of zinc consumed per second (by voltaic action) when one ampere flows through the cell, then Iz is the number of grams of zinc consumed per second when I amperes flows through the cell. Let j be the number of joules of energy developed by the consumption of one gram of zinc, then jIz joules of energy is developed per second by the consumption of Iz grams of zinc per second. *Let us assume that all of this energy is used in the maintenance of the current* I *which is flowing through the cell.* Then the battery does work at the rate of jIz joules per second in maintaining the current. Let the resistance of the circuit be R ohms; then heat is generated in the circuit at the rate of RI^2 joules per second, and, evidently, we must have

$$jIz = RI^2$$

or

$$I = \frac{jz}{R}$$

from which it is evident that the product of the two factors j and z, as defined above, is equal to what we call the electromotive force of the battery. See equation (13*b*).

42. Polarization of a battery.—If the electromotive force of a battery were invariable, then the current delivered by the battery would be doubled by reducing the resistance of the entire circuit* to one half, according to Ohm's law. The current delivered by a battery is not doubled, however, when the resistance of the circuit (the entire circuit) is halved, because the electromotive force of a battery falls off more or less with continued flow of

* Including the circuit of wire and the electrodes and electrolyte in the battery itself.

current, or when the flow of current is greatly increased. When a battery delivers current the chemical action quickly exhausts the electrolyte (the acid or salt solution) in the immediate neighborhood of the electrodes (carbon and zinc plates), the energy of the chemical action is reduced, and the battery is weakened. This weakening shows itself as a decrease of electromotive force, and it is called *polarization.*

The gravity Daniell cell does not polarize to any considerable extent. The ordinary dry cell polarizes greatly.

43. Ohm's law and Joule's law are nearly always applied to a portion of an electric circuit, not to an entire electric circuit.—

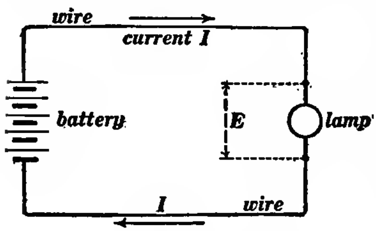


Fig. 53.

Consider the electric lamp in Fig. 53. Let R be the resistance of the lamp in ohms, and let I be the current flowing in the circuit in amperes. Then RI^2 is the rate in watts at which heat is generated in the lamp.

RI $(= E)$ is the electromotive force between the terminals of the lamp.

EI $(= RI^2)$ is the rate at which energy is delivered to the lamp.

These statements all refer *to the lamp* in Fig. 53, not to the entire circuit.

To avoid confusion one should always speak of the current **IN** a circuit; of the resistance **OF** a circuit (or the resistance of a portion of the circuit); and of the electromotive force or potential difference **BETWEEN THE TERMINALS OF** any portion of a circuit.

44. Definition of the volt.—Consider any portion of an electric circuit, for example consider the lamp in Fig. 53, and let I be the current flowing in the circuit. Then the electromotive force E between the terminals of the lamp is equal to RI as stated in the previous article, and if R is expressed in ohms and I in

amperes, then E $(= RI)$ is expressed in *volts*. That is, the product *ohms* $\times$ *amperes* gives *volts*.

One volt is the electromotive force between the terminals of a one-ohm resistance when a current of one ampere is flowing through the resistance.

One abvolt is the electromotive force between the terminals of a resistance of one abohm when a current of one abampere is flowing through the resistance. One volt is equal to 10^8 abvolts.

The electromotive force of an ordinary gravity cell is about 1.1 volts. The electromotive force of an ordinary dry cell is about 1.5 volts. The voltages commonly used for electric lighting and motor service are 110 volts and 220 volts; that is to say, the voltage between the supply wires in a building is usually either 110 volts or 220 volts. The usual voltage for electric railway service is 500 volts; that is to say, the voltage between the trolley wire and the rails is generally about 500 volts.

45. The direct-current voltmeter.—Consider an ammeter (see Art. 18) of which the resistance is R ohms. When a current of I amperes flows through the ammeter the electromotive force across the terminals of the instrument is RI volts, *and the scale of the instrument can be numbered so as to give the value of* RI *in volts instead of giving the value of* I *in amperes.* An ammeter arranged in this way is called a *voltmeter*.

It would seem from the above that the only difference between an ammeter and a voltmeter would be in the numbering of the scale; but an instrument which is to be used as an ammeter must have a very low resistance in order that it may not obstruct the flow of current in the circuit **IN** which it is connected, and an instrument which is to be used as a voltmeter must have a very high resistance in order that it may not take too much current from the supply mains **BETWEEN** which it is connected. Thus a good ammeter for measuring up to 100 amperes has a resistance of about 0.001 ohm so that one-tenth of a volt would be sufficient to force the full current of 100 amperes through the instrument. A good voltmeter for measuring up to 150 volts has a resistance

of about 15,000 ohms, so that about 0.01 ampere would flow through the instrument if it were connected across the terminals of a 150-volt generator.

46. Measurement of power by ammeter and voltmeter.—The power delivered by a battery or dynamo (direct-current dynamo) is equal to EI watts, where E is the electromotive force between the terminals of the battery or dynamo in volts and I is the current in amperes delivered by the battery or dynamo.

The power delivered to a lamp (or in general to any portion of

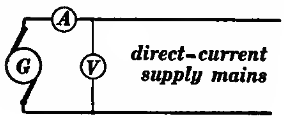


Fig. 54.
Ammeter and voltmeter connected for measuring power output of generator.

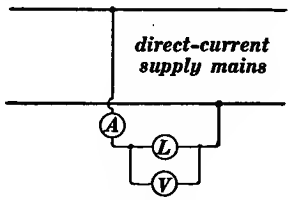


Fig. 55.
Ammeter and voltmeter connected for measuring power delivered to L.

a circuit) is equal to EI watts, where E is the electromotive force between the terminals of the lamp in volts, and I is the current in amperes flowing through the lamp.

Note.—Power cannot be measured by an ammeter and a voltmeter arranged as in Figs. 54 and 55 in an alternating-current system.

PROBLEMS.

49. A motor takes 79.78 amperes of current from 110-volt mains, and the motor-belt delivers 10 horse-power. What is the efficiency of the motor?

Note.—Power output of motor in watts divided by power intake of motor in watts gives the efficiency of the motor.

50. A so-called "25-watt, 110-volt" tungsten lamp takes 25 watts of power when it is connected to 110-volt supply mains. How much current does the lamp take, and what is the resistance of the lamp filament while the lamp is burning?

51. A motor to deliver 10 horse-power has an efficiency of 89 per cent. The motor is supplied with current at 110 volts across its terminals. Find the full-load current of the motor.

52. When electrical energy costs 11 cents per kilowatt-hour how much does it cost to operate for 10 hours a lamp which takes 0.227 ampere from 110-volt supply mains?

53. Find the cost of energy for operating a 5 horse-power motor at full load for 10 hours, the efficiency of the motor being 85 per cent and the cost of energy being 6 cents per kilowatt-hour.

54. When a certain dynamo electric generator is delivering no current it takes 1.75 horse-power to drive it. When the generator delivers 150 amperes it takes 25 horse-power to drive it. Calculate the electromotive force of the generator on the assumption that all of the additional power required to drive it is used to maintain the current of 150 amperes.

55. A coil of wire of which the resistance is to be determined is connected to 110-volt direct-current supply mains in series with an ammeter and a suitable rheostat, and a voltmeter is connected across the terminals of the coil. The ammeter reads 13 amperes and the voltmeter reads 80.6 volts. What is the resistance of the coil?

56. Assume the actual cost of electrical energy delivered to a street car to be 0.8 cent per kilowatt-hour. Find the cost of developing 100,000 British thermal units for heating the car first by an electrical heater in which all of the heat generated is available for heating, and second by burning coal costing $8 per ton (2,000 pounds) and giving 14,000 British thermal units per pound of which 30 per cent, say, is lost by incomplete combustion and by flue-gas losses.

57. One cubic foot of good illuminating gas costing one tenth of a cent gives about 600 British thermal units when it is burned, and about 20 per cent of the heat of a burner is taken up by the water in a tea kettle. On the other hand about 70 per cent of the heat given off by an ordinary electrical heater is given to a

tea kettle which completely covers the hot disk of the heater, and electrical energy for domestic use costs, say, 10 cents per kilowatt-hour. What is the cost of bringing two gallons of water (16.1 pounds or 7570 grams) from 0° C. to 100° C. by a gas burner and what is the cost by electric heater?

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47. Voltage drop in a generator.—Let I be the current in amperes which is being delivered by a battery (or dynamo), and *let R be the resistance of the battery in ohms.* Then a portion of the total electromotive force of the battery is used to force the current through the battery itself. The portion so used is equal to RI according to Ohm's law. If the total electromotive force of the battery is E volts, then the electromotive force between the terminals of the battery will be $(E - RI)$ volts. A voltmeter connected to the battery terminals would indicate E volts when the battery is delivering no current,* but the voltmeter would indicate $(E - RI)$ volts at the instant the battery begins† to deliver a current of I amperes. The electromotive force RI which is used to overcome the resistance of a battery (or dynamo) is called the *voltage drop* in the battery (or dynamo).

48. Voltage drop along a transmission line.‡—A current of I amperes is delivered to a distant motor or to a distant group of lamps over a pair of wires, *the combined resistance of the pair of wires being R ohms.*—Let E_0 be the voltage across the generator, and let E_1 be the voltage across the motor or lamps as shown in Fig. 56. Then E_1 is less than E_0, the difference $(E_0 - E_1)$ is the electromotive force which is used to overcome

* Of course the battery delivers current to the voltmeter, but this is a negligible current because the resistance of the voltmeter is very large as compared with the resistance of the battery.

† Continued flow of current causes a decrease of voltage by polarization as explained in Art. 42.

‡ The electromotive force across the terminals of a battery or between any two given points is frequently spoken of as an electrical "*pressure difference*" or *potential difference*, and the voltage drop along a transmission line is frequently called the "*pressure drop* or *potential drop* along the line.

the resistance of both wires, and it is equal to RI volts. This loss of electromotive force along a transmission line is called the *voltage drop* along the line. For example, the electromotive force across the terminals of a generator is 115 volts. The generator supplies 100 amperes of current to a group of lamps at a distance of 1,000 feet from the generator, and the wire (2,000 feet of it)

Fig. 56.

which is used for the transmission line has a total resistance of 0.05 ohm. Therefore the voltage drop along the line is 100 amperes × 0.05 ohm, or 5 volts; and the voltage across the terminals of the group of lamps is 115 volts − 5 volts = 110 volts.

PROBLEMS.

58. A gravity Daniell cell of which the electromotive force is 1.07 volts and the resistance is 2.1 ohms is connected to a wire circuit of which the resistance is 5 ohms. (*a*) What current is produced? (*b*) What is the electromotive force between the terminals of the cell? (*c*) What is the electromotive force drop in the cell?

59. A voltmeter connected across the terminals of a set of 60 storage battery cells connected in series reads 120.4 volts when the battery is delivering no current, and the voltmeter reading falls instantly to 112.25 volts when the battery begins to deliver 15 amperes of current. What is the resistance of the battery?

Note.—When a battery continues to deliver current the voltage falls off because of polarization. The sudden drop of voltage at the instant that current delivery begins is due almost entirely to the battery resistance.

60. A voltmeter connected across the terminals of a battery reads 15 volts when the battery is not delivering current (except the negligible current which flows through the voltmeter), and the voltmeter reading drops suddenly to 9 volts when a wire

circuit having a resistance of 6 ohms is connected to the battery. What is the resistance of the battery?

61. A dynamo electric generator having an electromotive force of 115 volts between its terminals delivers 200 amperes to a group of glow lamps 1,000 feet distant from the generator. Find the diameter in mils of the copper wire required in order that 95 per cent of the power output of the generator may be delivered to the lamps.

Note.—If 95 per cent. of the power output of the generator is delivered to the lamps then the electromotive force between the mains at the lamps must be 95 per cent. of 115 volts, or 109.25 volts.

62. What size of copper wire is required to deliver current at 110 volts to a 10-horse-power motor of 85 per cent efficiency; the motor being 2,000 feet from the generator, and the electromotive force across the generator terminals being 125 volts?

63. A motor is to receive 100 kilowatts of power from a generator at a distance of 15 miles. A loss of 10 per cent of generator voltage (or 10 per cent of the generator output of power) is to be permitted in the transmission line. Find the generator voltage which must be provided for in order that copper transmission wires 200 mils in diameter may be used.

64. If a 10 per cent line loss is allowed as in the previous problem, but if the generator voltage is doubled, what size of copper transmission wires would be used to deliver 100 kilowatts at a distance of 15 miles?

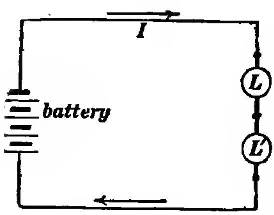

Fig. 57.
Two lamps in series.

49. Connections in series. — When two or more portions of an electric circuit are so connected that the entire current passes through each portion, then the portions are said to be *connected in series*. Thus Fig. 57 shows two lamps, L and L', connected in series. The ordinary arc lamps which are used to light city streets are connected in series, and the entire current delivered by the generator flows through each lamp; but the electromotive

force of the generator is subdivided. For example, a generator supplies 6.6 amperes at 2,000 volts to a circuit containing 30 arc lamps connected in series. The entire current, 6.6 amperes flows through each lamp, but the electromotive force across the terminals of each lamp is 1/30 of 2,000 volts or 67 volts. *The electromotive force of a generator is subdivided among a number of lamps or other units which are connected in series.*

50. **The voltmeter multiplying coil.**—Given a voltmeter which, for example, reads up to 10 volts; one can use such a voltmeter for measuring a higher voltage by connecting an auxiliary resistance in series with it. Thus Fig. 58 shows a voltmeter V

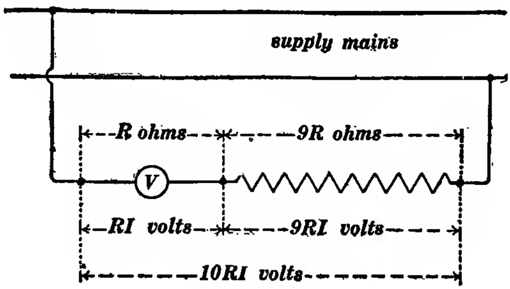

Fig. 58.

of which the resistance is R ohms, and which has an auxiliary resistance of $9R$ ohms connected in series with it. Under these conditions the voltage between the mains is found by multiplying the reading of the voltmeter by 10. This may be explained as follows: Let I be the current flowing through the circuit in Fig. 58. Then RI is the electromotive force drop across the terminals of the voltmeter, and $9RI$ is the electromotive force drop across the terminals of the auxiliary resistance. Therefore $RI + 9RI$ or $10RI$ is the electromotive force between the mains; but the voltmeter reading is the value of the electromotive force between its terminals, namely RI; therefore the electromotive force between the mains is ten times as great as the voltmeter reading.

51. **Connections in parallel.**—When two or more portions of an electric circuit are so connected that the current divides, part of it flowing through each portion, then the portions are said to be *connected in parallel.* Thus Fig. 59 shows two lamps L and L', connected in parallel. The ordinary glow lamps which are used for house lighting are connected in parallel between copper mains which lead out from the terminals of the generator; and (if the resistance of the mains is negligible) the full voltage of the generator acts on each lamp, but the current delivered by the generator is subdivided. For example, a 110-volt generator supplies 1,000 amperes to 2,000 similar lamps connected in parallel with each other between the mains. The full voltage of the generator acts on each lamp, but each lamp takes only 1/2000 of the total current. *The current delivered by a generator is subdivided among a number of lamps or other units which are connected in parallel.*

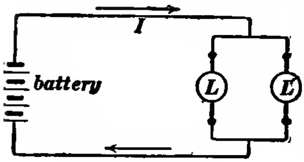

Fig. 59.
Two lamps in parallel.

Remark.—When a circuit divides into two branches, the branches are, of course, in parallel with each other, and either branch is called a *shunt* in its relation to the other branch.

52. **The division of current in two branches of a circuit.**—Figure 60 shows a battery delivering current to a circuit which branches at the points A and B. Let I be the current delivered by the battery, I' the current in the upper branch, I'' the current in the lower branch, R' the resistance of the upper branch, and R'' the resistance of the lower branch. The product $R'I'$ is the electromotive force between the branch points A and B, also the product $R''I''$ is

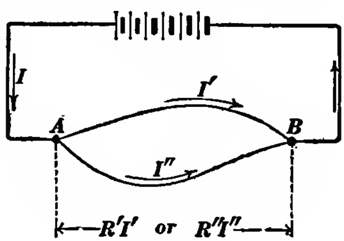

Fig. 60.

the electromotive force between the branch points A and B. Therefore we have:

$$R'I' = R''I'' \tag{i}$$

The current in the main part of the circuit is equal to the sum of the currents in the various branches into which the circuit divides. Therefore in the present case we have:

$$I = I' + I'' \tag{ii}$$

By using equations (i) and (ii) the values of I' and I'' can both be determined in terms of I, R' and R''.

It is important to note that a definite fractional part of the total current flows through each branch; and equation (i) shows that the currents I' and I'' are inversely proportional to the respective resistances R' and R''. Thus if R' is nine times as large as R'', then I'' is nine times as large as I'.

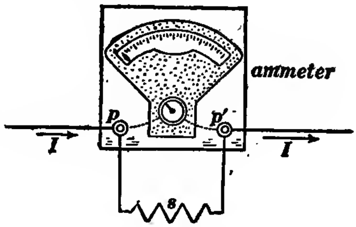

Fig. 61.
Impracticable arrangement of ammeter shunt.

53. The ammeter multiplying shunt.—A low-reading voltmeter can be used to measure a higher voltage by connecting an auxiliary resistance (a multiplying coil) in series with it as explained in Art. 50. A low-reading ammeter can be used to measure a larger current by connecting an auxiliary low lresistance (a multiplying shunt) in parallel with it.

It is not practicable, however, to use interchangeable shunts with a low resistance instrument (an ammeter). This may be illustrated by an example as follows: The ammeter in Fig. 61 has, let us say, a resistance of 0.01 ohm, and let us suppose that a 0.01 ohm shunt s is connected across its terminals. Under these conditions one half of the total current flows through the ammeter and one half flows through s. Therefore the value of the total current is twice the ammeter reading. The difficulty, however, is that if s is detachable there is likely to be an appre-

ciable* unknown resistance in the contacts of s with the two binding posts p and p' so that s may be in fact 10 or 20 per cent greater than it is supposed to be. *Any circuit* **in which** *binding-post contacts are to be made must be of fairly high resistance if the uncertain resistance at the contacts is to be negligible.*

Figure 62 shows an ammeter provided with a permanent shunt, j and j being soldered joints. In this case the shunt s may be once for all adjusted by the maker of the instrument so hat the full deflection of the instrument may correspond to any desired number of amperes. In fact a manufacturer usually makes the working part, C, of all of his ammeters alike. The only difference between an ammeter for large current and an ammeter for small current is in the resistance of the shunt s.

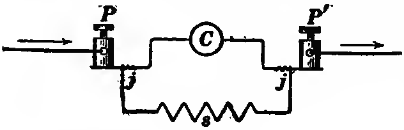

Fig. 62.
Practicable arrangement of permanent ammeter shunt.

54. Combined resistance of a number of branches of a circuit.—(a) The combined resistance of a number of lamps or other units *connected in series* is equal to the sum of the resistances of the individual lamps. (b) The combined resistance of a number of lamps or other units *connected in parallel* is equal to the reciprocal of the sum of the reciprocals of the resistances of indivi dual lamps. Proposition (a) is almost self-evident. Proposition (b) may be established as follows: Let E be the electromotive force between the points A and B where the circuit divides into a number of branches (see Fig. 60). Then, according to Ohm's law, we have:

$$I' = \frac{E}{R'} \tag{i}$$

$$I'' = \frac{E}{R''} \tag{ii}$$

$$I''' = \frac{E}{R'''} \tag{iii}$$

* Appreciable, that is, as compared with 0.01 ohm.

where R', R'' and R''' are the resistances of the respective branches, and I', I'' and I''' are the currents flowing in the respective branches.

Let I be the total current flowing in the circuit $(= I' + I'' + I''')$. *The combined resistance of the branches is* **defined** *as the resistance through which the electromotive force* E *between the branch points would be able to force the total current* I. That is, the combined resistance is defined by the equation:

$$I = \frac{E}{R} \qquad \text{(iv)}$$

in which R is the combined resistance. Adding equations (i), (ii), and (iii), member by member, and substituting E/R for $I' + I'' + I'''$, we have

$$\frac{E}{R} = \frac{E}{R'} + \frac{E}{R''} + \frac{E}{R'''} \qquad \text{(v)}$$

whence

$$R = \frac{1}{\frac{1}{R'} + \frac{1}{R''} + \frac{1}{R'''}} \qquad (14)$$

55. **Wheatstones's bridge.**—Four resistances x, R, a and

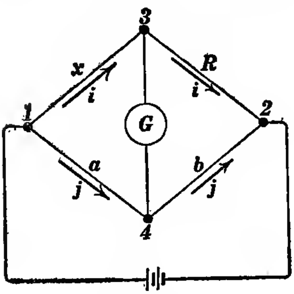

Fig. 63*a*.

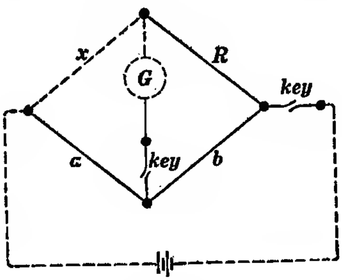

Fig. 63*b*.

b, Fig. 63, are adjusted so that the galvanometer G gives no deflection (no current through the galvanometer). Then

$$\frac{a}{b} = \frac{x}{R} \quad \text{or} \quad x = \frac{aR}{b} \qquad \text{(i)}$$

from which x can be calculated if R and the ratio a/b are known.

Proof of equation (*i*).—There being no current through G in Fig. 63*a*, the same current i flows through x and R, and the some current j flows through a and b. Also since there is no current through G the electromotive force between points 3 and 4 is zero. Therefore the electromotive force between 1 and 3 (which is equal to xi) must be equal to the electromotive force between 1 and 4 (which is equal to aj) so that

$$xi = aj \tag{ii}$$

Similarly, the electromotive force Ri between 3 and 2 must be equal to the electromotive force bj between 4 and 2, so that

$$Ri = bj \tag{iii}$$

Whence, dividing equation (ii) by equation (iii), member by member we get equation (i).

The usual arrangement of Wheatstone's bridge, the so-called *box bridge*, is indicated in Fig. 63*b*, in which the dotted lines show connections outside of the box, x being the unknown resistance which is to be measured. The two resistances a and b are called the *ratio arms*, and usually each consists of 1-ohm, 10-ohm, 100-ohm and 1,000-ohm coils; and R is a *rheostat*, consisting of units, tens, hundreds and thousands of ohms. The unknown resistance x is connected as indicated in Fig. 63*b*, the ratio arms a and b are chosen, the rheostat resistance R is adjusted, until the galvanometer gives no deflection, and the value of x is then given by equation (i).

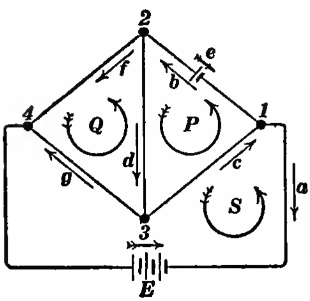

Fig. 64.

56. Kirchhoff's rules.—The equations which express the current and voltage relations in any complicated network of conductors are most easily formulated by the use of two rules which are known as Kirchhoff's rules.

Figure 64 shows a net-work of conductors in which are connected two batteries of which the electromotive forces E and e act in the directions of the arrows E and e.

The currents in the various branches of the net-work are

represented by the small letters a, b, c, d, f and g; and the arrows indicate the *arbitrarily chosen directions* in which the respective currents are to be considered positive. If we should find ultimately that a, for example, is -1.6 amperes it would mean that the current a actually flows in the direction opposite to the arrow a.

The resistances of the various branches of the net-work are represented by the capital letters A, B, C, D, F and G. For example A is the resistance of the branch 1 to 4 including the resistance of the battery E, and B is the resistance of the branch 1 to 2 including the resistance of the battery e.

Kirchhoff's first rule. Current equations.—The algebraic sum of the currents flowing towards* each branch point is zero.

Therefore we get:

$$\text{for point 1:} \quad -a - b + c = 0 \qquad \text{(i)}$$

$$\text{for point 2:} \quad +b - d - f = 0 \qquad \text{(ii)}$$

$$\text{for point 3:} \quad -c + d - g = 0 \qquad \text{(iii)}$$

and there is no need for writing down the current equation for point 4 because it is an equation which may be derived from (i), (ii) and (iii). If there are n branch points there are only $(n - 1)$ independent current equations.

Kirchhoff's second rule. Voltage equations.—The algebraic sum of the RI voltage drops around any mesh of the net work is equal to the algebraic sum of the battery voltages in that mesh, due attention being given to directions of arrows a, b, c, d, f, g, e and E with respect to the direction of the curled arrow which shows the chosen direction around the mesh. This matter will be understood by comparing the following equations with Fig. 64.

$$\text{for mesh } P \text{ we get:} \quad + Bb + Dd + Cc = -e \qquad \text{(iv)}$$

$$\text{for mesh } Q \text{ we get:} \quad - Dd + Ff - Gg = 0 \qquad \text{(v)}$$

$$\text{for mesh } S \text{ we get:} \quad - Aa - Cc + Gg = +E \qquad \text{(vi)}$$

Let us suppose that the two electromotive forces E and e, and all of the resistances are given. Then equations (i) – (vi) enable the calculation of the currents.

* Or away from.

PROBLEMS.

65. A telegraph line 80 miles long has a ground return (which we will assume to have negligible resistance), and the telegraph wire is, let us say, pure iron 0.100 inch in diameter. The telegraph instruments which are connected in circuit with the line have a combined resistance of 100 ohms. The line is operated by gravity cells each having an electromotive force of 1.07 volts and an internal resistance of 2 ohms. A current of 10 milliamperes is required to operate the line. How many gravity cells (connected in series) are required?

66. A millivoltmeter has a resistance of 15.4 ohms. What resistance must be connected in series with the instrument so that the scale reading may give volts instead of millivolts?

67. Three lamps (or other units) are connected in series to 110-volt mains, the resistances of the lamps are 10 ohms, 8 ohms and 4 ohms respectively, find the voltage across the terminals of each lamp.

68. Twenty gravity cells each having an electromotive force of 1.07 volts and an internal resistance of 2 ohms are connected 5 in series × 4 in parallel, and the batteries so connected deliver current to a coil of which the resistance is 6 ohms. What is the current in the coil?

69a. Three resistances of 4, 4 and 2 ohms respectively are connected in parallel; and two resistances of 6 ohms and 3 ohms respectively are connected in parallel. The first combination is connected in series with the second combination, and to a battery of negligible resistance and of which the electromotive force is 3 volts. What is the current in the 2 ohm resistance and what is the current in the 3 ohm resistance?

69b. A 3-ohm resistance and a 1.5-ohm resistance are connected in parallel, and this combination is connected in series with a 10-ohm resistance to a 2-volt battery cell of which the internal resistance is negligible. (*a*) Find the current in the 3-ohm resistance. (*b*) How much additional resistance must be connected in the circuit so that the current in the 3-ohm resistance

may be brought back to its original value after the 1.5-ohm resistance is cut or disconnected?

70. An ammeter has a resistance of 0.05 ohm. The instrument is provided with a shunt so that the total current through instrument and shunt is 10 times the current through the ammeter itself. What is the resistance of the shunt?

71. The scale of a direct-reading millivoltmeter has 100 divisions, each division corresponding to one thousandth of a volt between the terminals of the instrument. The instrument is connected to the terminals of a low-resistance shunt, and each division on the instrument scale corresponds to 0.25 ampere in the shunt. What is the resistance of the shunt?

72. A voltmeter which has a resistance of 16,000 ohms is connected in series with an unknown resistance R to 110-volt supply mains, and the reading of the voltmeter is 4.3 volts. What is the value of R?

73. A 40-mile telegraph line is disconnected from ground at both ends, the line is then connected to ground at one end through a 220-volt battery and a direct-reading voltmeter of which the resistance is 16,000 ohms, and the voltmeter reads 2.9 volts. What is the insulation resistance of the 40-mile telegraph line, and what is the insulation resistance of one mile of the line?

CHAPTER IV.

INDUCED ELECTROMOTIVE FORCE. ANOTHER ASPECT* OF THE MAGNETIC EFFECT OF THE ELECTRIC CURRENT.

There is no recognized unit of magnetic flux corresponding to the volt, the ampere, the ohm, etc. Therefore all quantities should be expressed in c.g.s. units (abamperes, abohms, abvolts, abhenrys, etc.) in every equation containing magnetic flux, or a reduction factor must be introduced as in equation (17).

57. **Back electromotive force in a motor armature.**—Before taking up the mathematical discussion of induced electromotive force it is highly desirable that the reader be brought face to face with induced electromotive force as an unmistakable physical fact, and the best way to do this is to consider the electric motor (direct-current type). *A much higher electromotive force is required to push a given current through the motor armature when it is running than when it is standing still.* Let the resistance of the armature from brush to brush be one ohm. Then 10 volts will maintain a current of 10 amperes through the armature when it is standing still as indicated in Fig. 65*a*, but a much higher electromotive force is required to maintain a current of 10 amperes through the running armature. Thus Fig. 65*b* shows

* Side push on the wires of a motor armature when current is flowing through the wires, and back electromotive force (induced electromotive force) in the wires when the motor is running are two aspects of one thing, and this electro-mechanical thing is closely analogous to the following purely mechanical thing: A man walks towards the center of the swinging span of a draw-bridge; the walking man necessarily exerts a side push which helps to turn the span, and the swinging span necessarily creates a back force on the man (a force which the man must overcome as he walks towards the center of the span).

The complete parallelism between the equations of electricity and magnetism on the one hand and the equations of mechanics on the other hand is a matter of very great interest, but it cannot be discussed profitably in an elementary text. This parallelism is brought out in any good discussion of *generalized coördinates.*

100 volts maintaining a current of 10 amperes through the running armature.*

It is harder, as it were, to push current through a running motor armature than to push current through the same armature while it is standing still. Something besides resistance must therefore oppose the flow of current through the running arma-

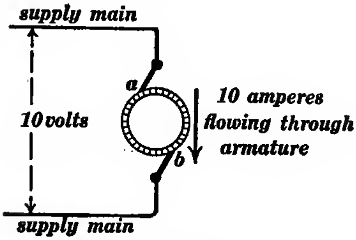

Fig. 65*a*.
Armature not running.

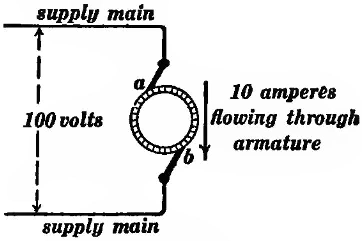

Fig. 65*b*.
Armature running.

ture. In fact a *back electromotive* force exists in the windings of the running armature. This back electromotive force is produced by the sidewise motion of the armature wires as they cut across the lines of force of the magnetic field in the gap spaces as shown by the fine lines in Fig. 36. An electromotive force produced in this way is called an *induced electromotive force.*

58. Expressions for induced electromotive force.—Each of the wires on the armature in Fig. 37 is in an intense magnetic field (this statement refers to the wires which are in the gap spaces between the armature core and the pole faces), each wire is at right angles to the lines of force of this intense field, and each wire is moving sidewise in a direction at right angles to itself and at right angles to the lines of force. This state of affairs is represented in Fig. 66; the wire bc slides on the two metal rails, moving sidewise at velocity v. Let I be the current in abamperes flowing around the circuit $abcd$ in Fig. 66. Then the side push exerted on bc by the magnetic field is lIH dynes according to equation (6) of Art. 21, where l is the length of the wire bc

* This effect must be actually shown by an experiment to be believed.

in centimeters and H is the intensity of the magnetic field in gausses. Therefore, neglecting friction, a force equal to F but in the direction of v must be exerted on the wire to make it move, this force will do work on bc at the rate Fv or at the rate $lIHv$ ergs per second, *and all of the work thus spent in moving the wire reappears as electrical work done by the induced electromotive force E in maintaining the current I.* Therefore* we must have $EI = lIHv$, whence

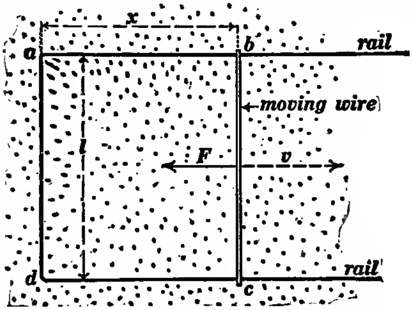

Fig. 66.
Dots represent magnetic lines of force which are perpendicular to the plane of the paper

$$E = lHv \tag{15}$$

that is to say, the electromotive force E in abvolts which is induced in the moving wire bc in Fig. 66 is equal to the product lHv, where l is the length of the moving wire in centimeters, H is the intensity of the magnetic field in gausses and v is the velocity of the moving wire in centimeters per second.

The electromotive force in abvolts which is induced in the moving wire bc in Fig. 66 is equal to the rate at which the wire cuts magnetic flux.—Consider the sidewise distance Δx moved by the wire bc in Fig. 66 during the short time interval Δt. Then:

$$\Delta x = v \cdot \Delta t \tag{i}$$

The area swept over by the wire during the time interval Δt is $l \cdot \Delta x$, and the amount of flux crossing this area is $\Delta\Phi = Hl \cdot \Delta x$, according to equation (4) of Art. 12, or, using the value of Δx from (i), we have:

$$\Delta\Phi = Hlv \cdot \Delta t \tag{ii}$$

and, of course, this is the amount of flux which is "cut" by the

* This fact was first pointed out by Helmholtz and it includes what has frequently been called Lenz's law.

wire during the time interval Δt. Therefore, dividing $\Delta\Phi$ by Δt we have the rate at which the wire cuts flux [in lines of force (or maxwells) per second], and from equation (ii) we have:

$$\frac{\Delta\Phi}{\Delta t} = Hlv \tag{iii}$$

That is, the rate at which the wire "cuts" flux is equal to Hlv, but Hlv is the electromotive force in abvolts induced in the moving wire, according to equation (15). *Therefore the electromotive force in abvolts induced in the moving wire bc in Fig.* 66 *is equal to the number of lines of force (maxwells) cut by the wire per second.*

The electromotive force in abvolts which is induced in the moving wire bc in Fig. 66 is equal to the rate of increase of the magnetic flux Φ through the circuit $abcd$ (across the rectangular area $abcd$).

The area of $abcd$ is lx square centimeters, and the magnetic flux Φ which passes through the opening $abcd$ is found by multiplying H by the area lx; that is:

$$\Phi = Hlx \tag{iv}$$

Now if x changes, Φ must change Hl times as fast, that is:

$$\frac{d\Phi}{dt} = Hl\frac{dx}{dt} \tag{v}$$

But, $\frac{dx}{dt}$ is the velocity v at which the wire moves sidewise. Therefore equation (v) becomes:

$$\frac{d\Phi}{dt} = Hlv \tag{vi}$$

Therefore, remembering that Hlv is the electromotive force induced in bc, and remembering that Φ is the magnetic flux through the opening $abcd$, we have the proposition as stated above.

59. Electromotive force induced in a stationary loop or coil of wire by a changing magnetic field or by a moving magnet.—

Article 58 refers to electromotive force induced in a wire which moves across a permanent or unchanging magnetic field. Let Φ be the amount of magnetic flux which passes through a loop of wire; then if anything whatever causes Φ to change, an electromotive force of E abvolts will be induced in the loop of wire such that

$$E = -\frac{d\Phi}{dt}$$

or, if there are Z turns of wire in the loop or coil, we will have

$$E = -Z\frac{d\Phi}{dt} \tag{16}$$

No attempt is here made to give a general derivation of this equation.

The negative sign is chosen in equation (16) for the following reason: Let us choose what we are to consider as the positive direction through the opening of a loop of wire, then the positive direction around the loop is conventionally taken as the direction in which a right-handed screw would have to be turned to travel in the chosen direction through the loop. Let us suppose that Φ is positive (the magnetic field which produces Φ would carry a north magnet pole in the positive direction through the loop), and let us suppose that Φ is increasing. Then $\frac{d\Phi}{dt}$ is positive *and experiment shows that the induced electromotive* E *is in the negative direction around the loop, that is,* E *is negative when* $\frac{d\Phi}{dt}$ *is positive.*

60. The fundamental equation of the direct-current dynamo. —The equation which expresses the electromotive force which is induced in the armature windings of a direct-current dynamo in terms of various data as explained below is called the fundamental equation of the dynamo. We can derive this equation from equation (15) of Art. 57, but it is instructive to carry out the argument from the beginning. Let us use c.g.s. units throughout, let us consider the dynamo as an electric generator, and let us think of the armature as rotating without energy losses of any kind* such, for example, as friction losses. *Then the mechanical power required to drive the dynamo armature is equal to the*

* This is not necessary but it avoids tedious qualifying specifications in the discussion.

electrical power output EI *of the armature*, where E is the electromotive force induced in the armature windings and I is the current delivered by the armature. Also let us limit the discussion to the simple form of dynamo with a bipolar field magnet and a ring-wound armature, as shown in Figs. 36–39.

Let r = radius of armature, measured out to the layer of wires.

L = length of armature core parallel to armature shaft. This is also the length of the pole faces parallel to the armature shaft.

b = breadth of each pole face measured along the circumference of the armature where the wires lie.

Z = number of wires on outside of armature. These wires are straight, they lie parallel to the armature shaft, and the length of the portion of each which lies in the gap space is L.

n = speed of armature in revolutions per second.

H = intensity of magnetic field in gap spaces. We assume this field to have the same intensity everywhere in the gap spaces, we assume the lines of force to be radial as shown in Fig. 36, and we ignore the fringe of the magnetic field which spreads out beyond the edges of the pole faces.

The total current I is supplied by the coming together at the brushes of $I/2$ abamperes flowing through the windings on each side of the armature. That is, the current in each armature wire is $I/2$ abamperes.

The side push of the magnetic field on each armature wire in the gap spaces opposes the motion of the armature, and is equal to $LHI/2$ dynes according to equation (6) of Art. 21; and the number of armature wires in the gap spaces at any time is $\frac{2b}{2\pi r} \times Z$. Therefore the total tangential drag or force on the armature wires is $\frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z$ dynes, and the power P

required to drive the armature against this dragging force is equal to the product of this force and the velocity $(2\pi rn)$ of the armature wires, that is $\frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z \times 2\pi rn$. But this power is equal to the power output EI as above explained. Therefore $EI = \frac{LHI}{2} \times \frac{2b}{2\pi r} \times Z \times 2\pi rn$, or

$$E = (LbH)Zn \tag{i}$$

But the area of a pole face Lb multiplied by H gives the amount of magnetic flux Φ which enters the armature core from the N-pole of the field magnet (and leaves the armature core to enter the S-pole of the field magnet), according to equation (4) of Art. 12. Therefore equation (i) becomes

$$E \text{ (in abvolts)} = \Phi Zn$$

or, since one volt is equal to 10^8 abvolts, we have

$$E \text{ (in volts)} = \Phi Zn \times 10^{-8} \tag{17}$$

Terminal voltage equation of direct-current generator.—Let R be the resistance of the armature of a dynamo, including brush contacts and brushes, and let I be the current delivered. Then RI is voltage drop in the armature, and the voltage E_b between brushes is $\Phi Zn \times 10^{-8}$ *minus* RI. That is

$$E_b = \Phi Zn \times 10^{-8} - RI \tag{18}$$

Speed equation of the direct-current motor.—Let E_b be the voltage across the brushes of the motor (supply voltage). Then E_b is used in part to overcome the resistance of the armature, the part so used is equal to RI; and E_b is used in part to overcome the back electromotive force $\phi Zn \times 10^{-8}$ Therefore $E_b = \Phi Zn \times 10^{-8} + RI$, and solving for n we get:

$$n = \frac{E_b - RI}{\Phi Z \times 10^{-8}} \tag{19}$$

A very interesting use of this equation is in the discussion of the speed characteristics of the direct-current shunt motor.*

* A very simple discussion of this matter is given in W. S. Franklin's *Elements of Electrical Engineering*, Vol. I, pages 196–200, published by Franklin and Charles, Bethlehem, Pa.

PROBLEMS.

74. The armature of a direct-current motor has a resistance of 0.064 ohm and when the motor is running under full load a current orf 81 amperes is forced through the armature from 110-volt supply mains. How much power is delivered to the motor armature? How much power is lost in heating the armature wires? What is the back electromotive force in the armature windings? How much power is expended in forcing the current of 81 amperes to flow against the back electromotive force? Where does this power go to?

75. The winding of an electromagnet has a resistance of 22 ohms, and when the winding is connected across 110-volt supply mains the current in the coil rises from zero at the beginning to 5 amperes ultimate value. The current must therefore pass in succession the values 1 ampere, 2 amperes, 3 amperes and 4 amperes. Consider the instant when the current is passing the value of 2 amperes, and calculate the values of the following quantities at this instant: (*a*) The rate at which work is being delivered to the coil, (*b*) The rate at which work is being spent in overcoming a back electromotive force in the coil due to the increasing magnetism of the rod, and (*d*) The value of the back electromotive force due to the increasing magnetism of the rod.

76. The winding of the electromagnet in the previous problem has 10,000 turns of wire. Find how fast the magnetic flux through the winding is increasing as the current is passing the value of 2 amperes

77. A long slim magnet of which the strength of each pole is 1,500 units is placed through a coil containing 20,000 turns of wire, and jerked out of the coil so that the magnetic flux through the coil due to the manget drops to zero during 0.004 second. What is the average value during the 0.004 second of the induced electromotive force in the coil?

78. A simple two-pole, direct-current dynamo with a ring winding on its armature has its field winding connected to 110-volt supply mains so that its field excitation is constant. Under

these conditions the armature flux Φ is nearly constant (regardless of current through the armature), and *we will assume that* Φ *is strictly constant.* There are 560 wires $(= Z)$ on the outside of the armature, and when the armature is driven at a speed of 900 revolutions per minute the electromotive force induced in the armature is 110 volts as indicated by a voltmeter connected to the brushes (current through armature very small). What is the value of the armature flux Φ? The resistance of the armature between brushes is 0.6 ohms. At what speed will the dynamo run as a motor at full load (10 amperes flowing through armature) when the armature takes current from 110-volt supply mains?

79. A direct-current dynamo (shunt dynamo) operating as a generator is connected as indicated in Fig. 67, and, with 110 volts

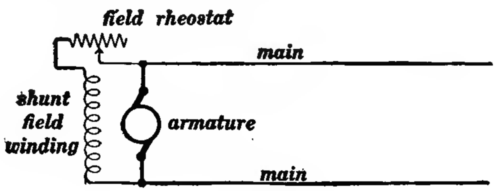

Fig. 67.
Shunt dynamo diagram.

across its armature terminals it delivers 200 amperes to the distributing mains. The resistance of the shunt field winding and rheostat is 11 ohms. How much current is delivered by the armature? What is the power output of the armature? How much power is expended in field excitation? How much power is delivered to the distributing mains?

Note.—After the current in the field winding and the "strength" of the field magnet become steady in value, all of the power delivered to the field winding reappears as heat in the winding in accordance with Joule's law. Therefore Ohm's law applies to the field winding. No power would be required to maintain the magnetism of the field magnet if a field winding of zero resistance could be obtained, although a zero-resistance field winding could not, of course, be connected as in Fig. 67. When, however, the field magnet *is being magnetized* (during a small fraction of a second at the beginning) then some of the power delivered to the field winding does not reappear as heat in accordance with Joule's law, but is used to establish the magnetism.

80. A thin copper disk r centimeters in radius is mounted on a spindle and driven at a speed of n revolutions per second. The spindle is mounted along the axis of a large and very long coil of wire (l centimeters long) having Z turns of wire distributed uniformly over its length, and a current of I abamperes flows through the coil. The rotating disk has a brush rubbing on its edge and another brush rubs on the tip end of the spindle. These two brushes are connected through a sensitive galvanometer to the treminals of a resistance R through which the entire current I flows. The disk is speeded up until the galvanometer gives no deflection. Make a diagram of the connections, and find an expression for R in terms of r, l, Z and n.

Note.—This problem refers to the celebrated arrangement which was devised by Lorentz for measuring resistance directly in terms of purely mechanical data.

61. The alternating-current dynamo.—The alternating-current dynamo is usually called an *alternator*. The following discussion may be taken as referring to the alternator as a *generator*. When the alternator is used as a motor it is called a *synchronous motor*.

Consider one of the straight wires on the armature in Fig. 37. The electromotive force which is induced in this particular wire is in *one* direction while the wire is sweeping across the north pole face and in the *opposite* direction while the wire is sweeping across the south pole face the field magnet, and if the two ends of this particular wire were to be kept permanently connected to the terminals of an outside circuit by means of sliding contacts, an alternating current would be delivered to the outside circuit.

This simple arrangement includes all of the essential features of the simple alternator. The actual arrangement of the simple alternator may be understood with the help of Figs. 68, 69 and 70. These figures show the inwardly projecting magnet poles of what are called *multipolar field magnets*. These field magnets are magnetized or *excited* by direct current from some outside source, generally a small direct-current generator which is called an *exciter*.

The large dotted circles in Figs. 68, 69 and 70 represent the ends of the cylindrical armature core, the inner circle represents the front end and the larger circle represents the back end en-

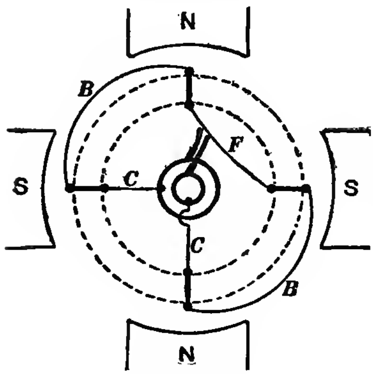

Fig. 68.

Possible armature winding diagram for 4-pole alternator.

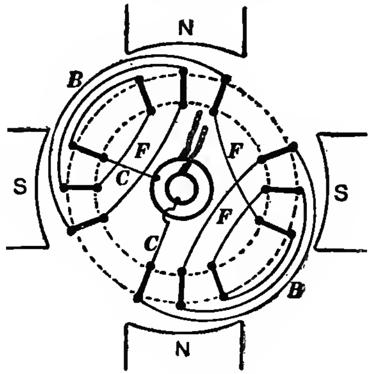

Fig. 69.

Possible armature winding diagram for 4-pole alternator.

larged so as to be seen. The short heavy radial lines represent the straight wires which lie on the surface of the armature core parallel to the armature shaft, the curved lines BB represent cross-connections on the back end of the armature, the curved lines FF represent cross-connections on the front end of the armature, and the lines CC represent connections to the two insulated metal rings which are called *collector rings*. Two carbon blocks or metal brushes rub on these collector rings thus maintaining permanent connections with the terminals of the armature winding, and the outside circuit to which alternating current is to be delivered is connected to the two carbon blocks or brushes.

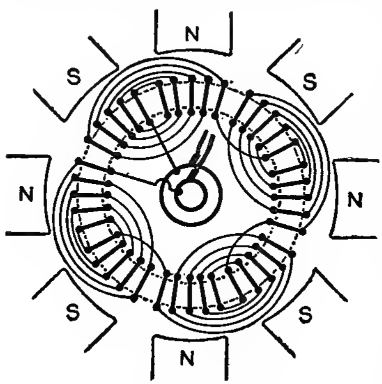

Fig. 70.

Possible armature winding diagram for 8-pole alternator.

The simple alternator above described is called a *single-phase alternator.* It has a single armature winding and two collector rings. The *two-phase alternator* has two distinct armature windings, each winding being connected to two collector rings (four collector rings in all). The *three phase-alternator* has three distinct armature windings, each winding being connected to two collector rings (six collector rings in all). Because of certain relations between the three distinct alternating currents which are delivered by the three armature windings of a three-phase alternator (delivered to three distinct receiving circuits, of course) it is possible to use only *three* collector rings; and this is the usual practice.

Definition of cycle. Definition of frequency.—The electromotive force of an alternator (and also the current delivered by an alternator) is subject to rapid reversals in direction, the electromotive force passes through a set of positive values as represented by the ordinates of the portion P of the curve in Fig. 71, and then through a similar set of negative values N,

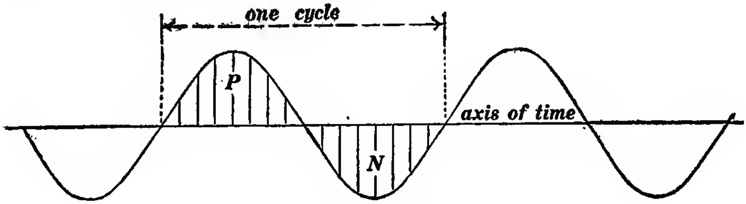

Fig. 71.

repeatedly. The two sets P and N together constitute a *cycle,* and the number of cycles per second is called the *frequency* of the alternating electromotive force or current.

The electromotive force which is induced in the armature winding which is shown in Fig. 68 passes though a set of positive values, let us say, while a given wire is sweeping across the face of a north pole of the field magnet, and through a set of negative values while the given wire is sweeping across a south pole face, so that there are p cycles per revolution of the armature where

p is the number of *pairs* of field magnet poles. Therefore the frequency $f = pn$ where n is the speed of the armature in revolutions per second.

PROBLEMS.

81a. Make a diagram like Fig. 68 and, assuming the armature to rotate in a clockwise direction, indicate by an arrow the direction of the electromotive force which is induced in each of the armature conductors (each short heavy radial line). Show which collector ring is positive and which is negative?*

81b. Make diagram like that called for in problem 81a but with the armature in Fig. 68 turned 90° forwards. Show which collector ring is positive and which is negative.

82. A disk of wood of radius r has Z turns of wire wound around its edge, and it is rotated n revolutions per second, about a diameter, in a uniform magnetic field of intensity H, the axis of rotation being at right angles to H. Derive an expression for the electromotive force induced in the coil.

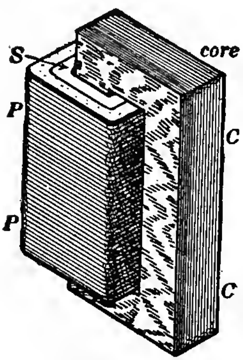

Fig. 72.
Alternating-current transformer.

62. The alternating-current transformer.—The alternating-current transformer consists of two separate windings of wire on a laminated† iron core as shown in Fig. 72. One of these windings is connected to an alternator and the rapid reversals of the magnetism of the core induce an alternating electromotive force in the other winding. The winding which receives alternating current from an alternator is called the *primary coil* and the other coil which delivers alternating current at a higher or lower voltage is called the *secondary coil*.

Step-down transformation.—A small alternating current may

* The positive terminal of a generator is the terminal at which current flows out of the generator to the receiving circuit.

† See Art. 64.

be delivered at high voltage to the coil-of-many-turns, in which case the coil-of-few-turns will deliver a large alternating current at low voltage. This constitutes what is called *step-down transformation.*

Step-up transformation.—A large alternating current may be delivered at low voltage to the coil-of-few-turns, in which case the coil-of-many-turns will deliver a small alternating current at high voltage. This constitutes what is called *step-up transformation.*

PROBLEMS.

83. A transformer has 100 turns of wire in one of its coils A and 1,000 turns of wire in its other coil B. Coil A is connected for a few thousandths of a second to 110-volt, direct-current, supply mains. Assuming resistance of coil A to be negligible, calculate rate of growth of magnetic flux through the transformer core and calculate electromotive force induced in coil B.

84. Let us assume that a negligibly small current in coil A, alone, or a negligibly small current in coil B, alone, will produce an indefinitely large magnetic flux Φ through the core of the transformer of the previous problem, then the magnetizing action (amperes $\times$ turns) of the current in one coil must always be equal and opposite to the magnetizing action (amperes $\times$ turns) of the current in the other coil. Why? Coil A is connected for a few thousandths of a second to 110-volt, direct-current, supply mains, and coil B is connected to a "non-inductive" receiving circuit of which the resistance is 2,155 ohms. The resistance of coil A is zero and the resistance of coil B is 45 ohms. Find the values of the following quantities during the few thousandths of a second. (*a*) Current delivered by coil B, (*b*) Current in coil A, and (*c*) Voltage across terminals of coil B.

Note.—A "non-inductive" circuit is one in which the current is always given by Ohm's law even at the instant that an electromotive force begins to act upon it.

85. Calculate the current which is delivered by coil B and the voltage across the terminals of B when the resistance of coil A is 0.4 ohm, everything else being as specified in the previous problem.

63. The induction coil.—An iron rod or core wound with insulated wire can be repeatedly magnetized and demagnetized by connecting a battery to the winding and repeatedly making and breaking the circuit; and the increase and decrease of magnetism of the core thus produced can be utulized to induce electromotive forces in an auxiliary coil of wire wound on the iron core. Such an arrangement is called an *induction coil.* The winding through which the magnetizing current from the battery flows is called the *primary coil,* and the auxiliary winding in which

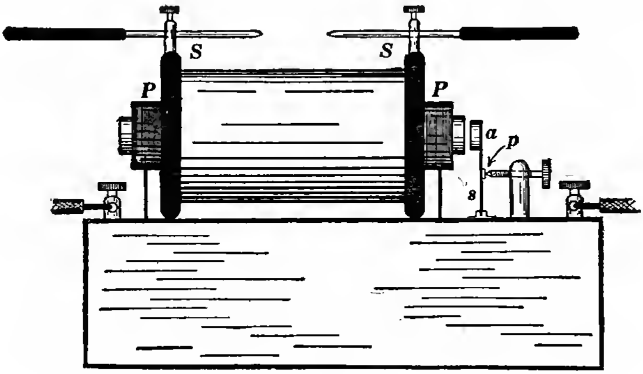

Fig. 73.
Induction coil.

the desired electromotive forces are induced is called the *secondary coil.* The iron core is always made of a bundle of fine iron wires or strips of sheet iron.

A general view of an induction coil is shown in Fig. 73, and the diagram of connections is shown in Fig. 74. When the iron core is magnetized, the block of iron *a* is attracted, and the battery circuit is broken at the point *p*. The iron core then loses its magnetism, and the spring *s* brings the points at *p* into contact again so that the battery current again flows through the circuit and magnetizes the iron core. The iron block *a* is then attracted again, and the above action is repeated.

When the iron core of an induction coil is magnetized, a

momentary pulse of electromotive force is induced in the secondary coil; and when the iron core is demagnetized a reversed momentary pulse of electromotive force is induced in the secondary coil. Electromotive forces are induced only while the core is being magnetized or demagnetized, and each pulse of electromotive force may be made very large in value (manv thousands

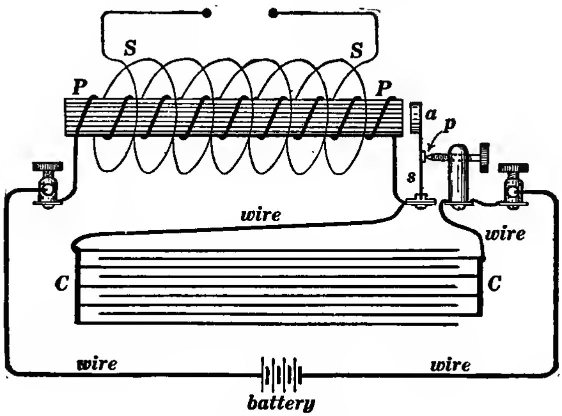

Fig. 74.
Induction coil diagram.

of volts) by using many turns of wire in the secondary coil and by providing for the quickest possible magnetization or demagnetization of the core. A battery cannot, however, magnetize a core very quickly when connected to a magnetizing coil; in fact a very considerable fraction of a second is required for the core to become magnetized. Therefore during the magnetization of the iron core of an induction coil the electromotive force induced in the secondary coil is a comparatively weak pulse of fairly long duration.

On the other hand the use of the condenser CC Fig. 74, causes the iron core of the induction coil to be demagnetized very quickly as explained in Art. 66, and this quick demagnetization induces in the secondary coil an intense pulse of electromotive force of very short duration.

The iron core of the alternating-current transformer should form a complete "magnetic circuit" as shown in Fig. 72, but the induction coil must have its iron core in the form of an open "magnetic circuit" as shown in Fig. 74, because after the core has been magnetized *it is the energy of this magnetism which becomes available when the primary circuit is broken*, and the greater part of the available energy of a magnet resides in the air field near the poles of the magnet.

64. Eddy currents. Lamination.—Figure 75 shows an end view of an iron rod surrounded by a wire ring. While the iron

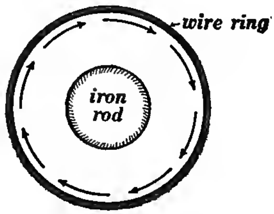

Fig. 75.

Current induced in wire ring.

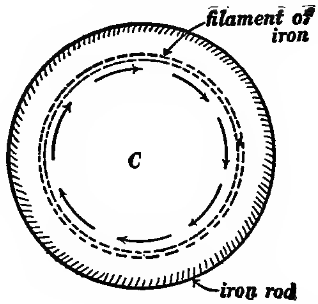

Fig. 76.

Current induced in filament of solid iron rod.

rod is being magnetized or demagnetized an electromotive force is induced in the ring, according to Art. 59, and an electric current is produced in the ring in the direction of the small arrows or in the opposite direction. Figure 76 shows the end of a larger iron rod. While the rod is being magnetized or demagnetized an electric current is produced in the circular filament of iron. *The increasing or decreasing magnetism of the central portion C of the rod in Fig.* 76 *has the same action on the filament of iron in Fig.* 76 *as the increasing or decreasing magnetism of the iron rod in Fig.* 75 *has on the wire ring in Fig.* 75.

Every circular filament in an iron rod has more or less current induced in it while the rod is being magnetized or demagnetized. Thus the currents which are induced in a solid iron rod while it is being magnetized or demagnetized are in the directions of the arrows in Fig. 77 or in the opposite directions, and these currents are called *eddy currents*. One effect of these eddy currents is to

make it impossible to magnetize or demagnetize a solid iron rod quickly and another effect is to generate heat in the rod.

If the rod is a bundle of thin strips of sheet iron, as shown in Fig. 78, or a bundle of fine iron wires, as shown in Fig. 79, then the eddy currents (as shown in Fig. 77) cannot flow because the strips of iron in Fig. 78 and the iron wires in Fig. 79 are suffi-

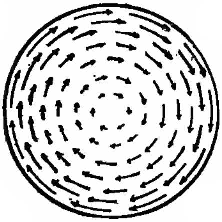

Fig. 77.
Currents induced in solid iron rod.

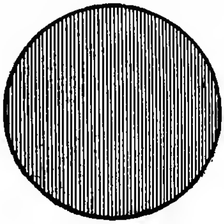

Fig. 78.

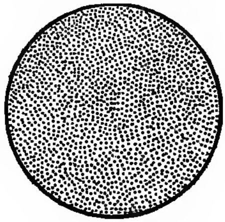

Fig. 79.

ciently insulated from each other by the thin coating of oxide which always covers the iron.

Eddy currents are not only produced in a solid iron rod while it is being magnetized or demagnetized, but eddy currents are also generally produced in a piece of solid iron (or in any solid piece of metal) which moves near a magnet. Thus if the cylinder *AA*, Fig. 36, were solid and if it were set rotating as indicated by the curved arrows in Fig. 37, eddy currents would be produced in it as indicated by the circles with dots and crosses in Fig. 80.

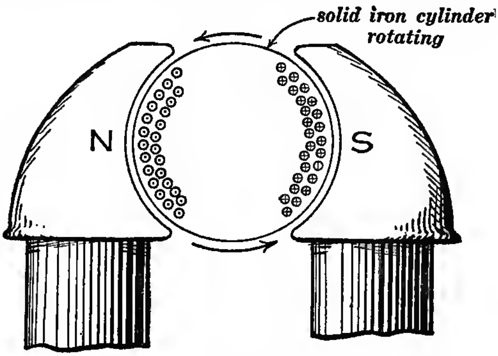

Fig. 80.
Eddy currents in rotating solid cylinder; away from reader on right side, towards reader on left side.

If the cylinder is built up of thin sheet iron disks or stampings, these eddy currents cannot flow because the disks are sufficiently insulated from each other by films of on oxide.

An iron rod or core which is built up of stampings of thin sheet iron or of fine iron wires is said to be *laminated.* Armature cores of dynamos and transformer cores are always laminated.

The damping disk of the watt-hour meter.—An interesting example of eddy currents is afforded by the damping disk of the watt-hour meter, a copper or aluminum disk which rotates between the poles of a horse-shoe magnet.

65. Inductance of a coil or circuit.—While a boat is being brought up to full speed a part of the propelling force is used to accelerate the boat (to cause its velocity to increase). Similarly, when a circuit is connected to supply mains (direct-current supply mains) some time elapses before the current reaches its steady value as given by Ohm's law, and during the whole of this time RI is less than the supply voltage E.* *Therefore during the short time that is required for the current to reach its final steady value a portion of the supply voltage E is used to cause the current in the circuit to grow.*

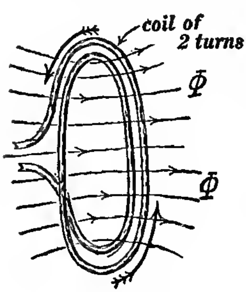

Fig. 81.

Figure 81 represents a coil of Z turns of wire connected to a battery (battery not shown). The current grows from zero to a final steady value as given by Ohm's law, and while the current is growing the magnetic flux Φ through the coil (the flux that is due to the current in the coil) grows. In the following discussion Φ is the *average flux per turn of wire,*† and Φ is at each instant proportional to the current i, that is to say, when i is doubled the value of Φ is doubled. Therefore we may write:

$$\Phi = ki \qquad \text{(i)}$$

* This is evident when we consider that $RI = E$ or $I = E/R$ when the current reaches its final steady value.

† There is more flux through the outer turns than through the inner turns, and Φ represents the average at any instant.

where k is a constant for the given coil. If i is growing it is evident that Φ must grow k times as fast, that is we must have

$$\frac{d\Phi}{dt} = k\frac{di}{dt} \tag{ii}$$

But the growing flux induces an electromotive force in the coil and this induced electromotive force E' is

$$E' = -Z\frac{d\Phi}{dt} \tag{iii}$$

according to equation (16). *Therefore to make the current grow an outside electromotive force (a portion of the electromotive force of the battery in Fig. 81) must be used to overcome* E', and the portion so used is equal and opposite to E' or equal to $+Z\frac{d\Phi}{dt}$. Therefore the electromotive force which must act on a circuit or coil to make the current in the circuit or coil grow is equal to $Z\frac{d\Phi}{dt}$ or to $kZ\frac{di}{dt}$. Let us use the letter L for the quantity kZ, that is

$$L = kZ \tag{iv}$$

Then we have

$$e = L\frac{di}{dt} \tag{20}$$

where e is the *part* of the electromotive force of the battery in Fig. 81 which is causing the current to grow, $\frac{di}{dt}$ is the rate of growth of the current in amperes per second (or in abamperes per second), and L is what is called the *inductance* of the coil or circuit.

The portion, f, of the propelling force which causes the velocity v of a boat to grow (increase) is $f = m\frac{dv}{dt}$. *The inductance* L *of a circuit is precisely analogous to mass in mechanics.* See Appendix D.

Remark.—If there is an iron core in the coil in Fig. 81, the flux Φ will be nearly proportional to i for small values of the current, but as the iron core approaches magnetic saturation Φ is no longer even approximately proportional to i. A coil with an iron core does not, therefore, have a definite inductance.

Flux -turns.—The quatity Φ in the above discussion is the average flux per turn of wire as stated, and the product ΦZ is *the number of linkages of lines of force and turns of wire*, and it is usually called, simply, *flux-turns*. Putting $k = L/Z$ from (iv) in (i), we get

$$\Phi Z = Li \tag{21}$$

Definition of inductance.—*The inductance of a circuit is defined by equation* (20), *and a circuit has unit of inductance when one unit of electromotive force will cause the current in the circuit to increase at the rate of one unit per second.*

A circuit has an inductance of one *henry* when one volt will cause the current in the circuit to increase at the rate of one ampere per second.

A circuit has an inductance of one *abhenry* when one abvolt will cause the current in the circuit to increase at the rate of one abampere per second. One henry equals 10^9 abhenrys.

Non-inductive circuit.—When the inductance of a circuit is negligibly small the circuit is said to be *non-inductive.* The inductance is neglibigly small when the electromotive force $L\dfrac{di}{dt}$ is very small in comparison with Ri.

Kinetic energy of the electric current.—The kinetic energy in joules which is associated with a current of I amperes in a circuit or coil of which the inductance is L henrys is

$$W = \tfrac{1}{2}LI^2 \tag{22}$$

The kinetic energy of a current in a coil resides in the surrounding magnetic field.

Imagine the circuit to have zero resistance, then if an electro-

motive force E acts on the circuit the current will grow in accordance with equation (20) so that $E = L\frac{di}{dt}$. Therefore $Ei = Li\frac{di}{dt}$. But Ei is the rate at which work is done on the circuit $\frac{dW}{dt}$, so that

$$\frac{dW}{dt} = Li\frac{di}{dt}$$

Integrating this equation from $i = 0$ to $i = I$ we get equation (22).

Differential equation of growing current.—A constant electromotive force E acts on a circuit. Let i be the value of the current at any instant before the final steady value (E/R) of the current is established. Then a portion of E is used to overcome resistance, and the portion so used is equal to Ri; and the remainder of E is used to make the current grow, and this remainder is therefore equal to $L\frac{di}{dt}$, according to equation (20). Therefore we have

$$E = Ri + L\frac{di}{dt} \tag{23}$$

Integrating* this equation subject to the condition $i = 0$ when $t = 0$ (which means that time is reckoned from the instant that the constant electromotive force E begins to act on the circuit), we get

$$i = \frac{E}{R} - \frac{E}{R}\epsilon^{-(R/L).t} \tag{24}$$

where ϵ is the Naperian base (2.7183). Equations (23) and (24) apply to a circuit such as is shown in Fig. 81.

Differential equation of decaying current.—If the electromotive force [electromotive force E in equation (23)] ceases to act on the circuit, that is, if the battery in Fig. 81 is short

* A full discussion of this integration, and of the integration of equation (25), is given in Franklin, MacNutt and Charles' *Calculus*, pages 176–181; published by Franklin and Charles, Bethlehem, Pa., 1913.

8

circuited by a heavy copper connection between its terminals, then $E = 0$, and equation (23) becomes

$$0 = Ri + L\frac{di}{dt} \tag{25}$$

which is the differential equation of decaying current.

Integrating equation (25) ,we get

$$i = I\epsilon^{-(R/L)\cdot t} \tag{26}$$

where I is the value of the current when $t = 0$, and ϵ is the Naperian base.

Calculation of inductance* of a coil which consists of a single layer of wire wound on a long cylindrical wooden rod.—This is the only case in which a simple formula can be established for calculating L. The magnetic field intensity inside of the coil is $4\pi zI$, where z is the number of turns of wire per centimeter of length of coil and I is the current in the coil in abamperes† (see Art. 23). Therefore $\pi r^2 \times 4\pi zI$ is the flux through the opening of the coil where r is the radius of the coil, and $zl \times \pi r^2 \times 4\pi zI$ is the flux-turns due to current I where l is the length of the coil and zl is the total number of turns of wire in the coil. Therefore from equation (21) we have

$$\Phi Z = LI = 4\pi^2 r^2 z^2 lI$$

whence

$$L = 4\pi^2 r^2 z^2 l \tag{27}$$

This equation gives L in abhenrys; to reduce to henrys divide by 10^9.

PROBLEMS.

86. A coil of wire has a resistance of 20 ohms and an induc-

* The calculation of inductance is in general very complicated. See Formulas and Tables for Calculation of Inductance, by E. B. Rosa and F. W. Grover, *Bulletin of the Bureau of Standards*, Vol. 8, No. 1, 1911. An important and interesting discussion of this subject is given by S. Butterworth, in the *Philosophical Magazine* for April 1915. The absolute measurement of inductance is discussed by Rosa and Grover in the *Bulletin of the Bureau of Standards*, Vol. 1, No. 2, 1905.

† In all equations involving magnetic field intensity or magnetic flux c.g.s. units should be used (abamperes, abvolts etc.), because there are no recognized units of field intensity and flux which correspond to the ampere, volt, ohm, henry, etc.

ance of 0.6 henry. When connected to 110-volt direct-current supply mains the current rises to a steady value of 5.5 amperes. How fast does the current start to increase in the coil when it is first connected to the 110-volt supply mains and how long would it take for the current to reach its steady value of 5.5 amperes if it continued to increase at this initial rate?

Note.—The time which would be required for the current to reach its final steady value at its initial rate of increase is called the *time constant* of the circuit.

87. From the definition or the time constant of a circuit as given in the note to the previous problem show that the time constant of any circuit is equal to L/R where L is the inductance of the circuit and R is the resistance of the circuit.

88. Find the rate of growth of current in the coil of problem 86 at the instant that the current is passing the value of 3 amperes.

89. The choke coil of a lightning arrester consists of 40 turns of wire in a cylindical one-layer coil 10 centimeters in diameter and 30 centimeters long. Calculate the approximate inductance of the coil in henrys.

At the instant of a lightning stroke an electromotive force of 40,000 volts acts across the terminals of this coil. How fast does the current begin to grow in the coil?

90. A current is left to die away in a circuit of which the resistance is 10 ohms and the inductance is 0.05 henry. How fast is the current decreasing as it passes the value 4 amperes?

91. The coil of problem 86 is connected to 110-volt direct-current supply mains. What is the value of the growing current after 0.02 second?

91. An alternating current i flows through a circuit of which the inductance is L and of which the resistance is negligible. The expression for i is $i = I \sin \omega t$ where I and ω are constants. Find an expression for the electromotive force which must act on the circuit to cause the current to change in the prescribed manner.

CHAPTER V.

ELECTRIC CHARGE AND THE CONDENSER.

66. The elimination of the water hammer effect by an air cushion. The elimination of the spark at break by a condenser. —The water hammer effect which is produced when a hydrant is suddenly closed is sometimes sufficiently intense to burst the pipe or injure the valve of the hydrant. In some cases, therefore, it is desirable to protect the hydrant by an air cushion as indicated in Fig. 82. When the hydrant in Fig. 82 is closed (however quickly) the moving water in the pipe is brought to rest gradually as it slowly compresses the air in the chamber CC.

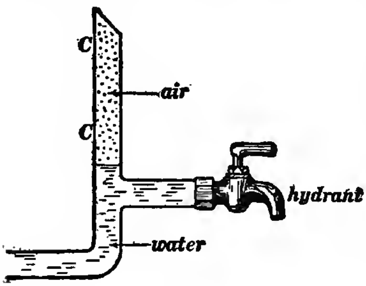

Fig. 82.

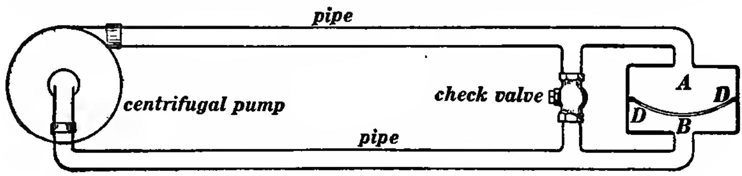

Fig. 83*a*.

Figure 83*a* shows a centrifugal pump maintaining a stream of water through a circuit of pipe. If the check valve is sudednly closed, the water will continue for a short time to flow through the circuit into the chamber A and out of the

Figure 83*b* shows a battery maintaining a "current of electricity" through a circuit. If the circuit is broken at p, the electric current will continue for a short time to flow through the circuit into the metal plate A and out of the metal plate

chamber *B*. This continued flow of the water into chamber *A* and out of chamber *B* causes a bending (a mechanical stress) of the elastic diaphragm *DD*, and this bending soon stops the flow of water; then the diaphragm unbends and produces a reversed surge of water through the circuit of pipe.

B. This continued flow of the electric current into plate *A* and out of plate *B* causes what we may think of as an "electrical bending" (an electrical stress) of the layer of insulating material *DD*, and this "electrical bending" soon stops the flow of current; then the layer of insulating material "unbends" and produces a reversed surge of electric current through the circuit.

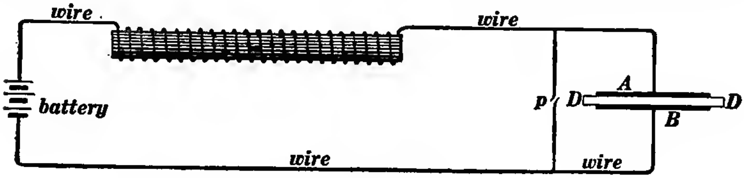

Fig. 83*b*.

The two metal plates *A* and *B*, Fig. 83*b*, together with the layer of insulating material between them constitute what is called a *condenser*. A condenser is usually made of sheets of

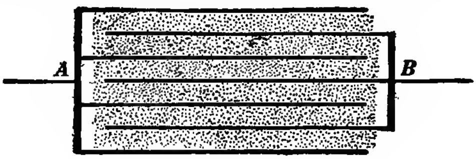

Fig. 84.

tin foil separated by sheets of waxed paper. Thus the heavy horizontal lines in Fig. 84 represent sheets of tin foil, and the fine dots represent insulating material. In order that the following effects may actually be observed the condenser in Fig. 83*b* must be made of a large number of sheets of tin foil and waxed paper.

The actual flow of current into the metal plate A and out of the metal plate B when the circuit is broken at p in Fig. 83*b* is shown by the fact that no spark at break is produced when the condenser AB is connected, whereas a very perceptible spark at break is produced when the condenser AB is not connected.

The reversed surge of current which takes place after the original current has been stopped in Fig. 83*b* may be shown as follows: Disconnect the condenser AB, make and break contact at p, hold a magnetic compass near one end of the core of the inductance* coil, and the core will be found to have retained a large portion of its magnetism; in other words, the core will not have become by any means completely demagnetized when the circuit is broken and the current reduced to zero. Then connect the condenser AB, make and break the circuit at p as before, and again test the core of the inductance coil with a compass. The core will now be found to have lost nearly the whole of its magnetism because of the reversed surge of current.

A reversed surge of current from the condenser in Figs. 73 and 74 is the cause of the very quick demagnetization of the core of an induction coil.

67. The momentary flow of current in an open circuit. Idea of electric charge.—When the metallic contact at p in Fig. 83*b* is broken the electric circuit remains closed as long as the current continues to flow across the break in the form of a spark. The intensely heated air in the path of a spark is a conductor. When the condenser AB is connected as shown in Fig. 83*b*, there is no spark at break, and the circuit is actually opened at the moment the contact at p is broken. The continued flow of current through the circuit after the contact at p is broken *is an example of the momentary flow of current in an open circuit.* The current continues for a very short time to flow through the open circuit into plate A and out of plate B after the circuit is broken at p, and *the two plates A and B are said to become electrically charged.*

68. Definition of the coulomb or ampere-second.—An electric

* A coil which has *inductance.* Do not confuse this term with *induction coil.*

current in a wire may be looked upon as the transfer of "electricity" along the wire, and the quantity Q of "electricity" which flows past a point on the wire during t seconds may be defined as the product of the strength of the current and the time, that is we may write:

$$Q = It \tag{28}$$

in which I is the strength of the current in amperes, and Q is the quantity of electricity which flows past a point on the wire during t seconds. It is evident from equation (28) that the product of amperes and seconds gives quantity of electricity, and therefore the unit of quantity of electricity is most conveniently taken as one *ampere-second*, meaning the amount of electricity which during one second flows past a point on a wire in which a current of one ampère is flowing. The ampere-second usually called the *coulomb*. One *ampere-hour* is the quantity of electricity carried in one hour by a current of one ampere.

The c.g.s. unit of charge* in the "electromagnetic system" is the amount of charge carried in one second by a current of one abampere, and it is called the *abcoulomb*. The abcoulomb is equal to 10 coulombs.

69. Electrostatic attraction. The electrostatic voltmeter.— When a momentary current flows into plate A and out of plate B in Fig. 83*b*, the plates are said to become electrically charged, as stated above, *the plate into which the momentary current flows is said to become positively charged and the plate out of which the momentary current flows is said to become negatively charged.* **Two plates which have thus been oppositely charged attract each other when the intervening insulating material is a fluid like air or oil.**

This attraction between two oppositely charged metal plates is utilized in the *electrostatic voltmeter* which consists of a very delicately suspended metal plate and a stationary metal plate, both carefully insulated. The electromotive force to be measured is connected to these plates, a momentary flow of current charges

* The word *charge* as here used means quantity of electricity.

one plate positively and the other plate negatively, the suspended plate is moved by the electrostatic attraction between the plates, and a pointer attached to the movable plate plays over a divided scale.

The c.g.s. unit of charge in the "electrostatic system" is that amount of charge which if concentrated on a very small body would repel a similar charge with a force of one dyne at a distance of one centimeter. The "electrostatic system" system of units is not used in this text.

70. Measurement of electric charge. The ballistic galvanometer.—A very large charge of electricity may be determined by observing the time during which the charge will maintain a sensibly constant measured current. Thus, a given storage cell can maintain a current, say, of ten amperes for eight hours so that the discharge capacity of the storage battery is equal to eighty ampere-hours. The quantities of charge which are most frequently encountered in the momentary flow of electric current in open circuits are, however, exceedingly small. For example, the terminals of a given condenser are connected to 110-volt direct-current supply mains, and the momentary flow of current represents the transfer of, say, 0.0001 of a coulomb which corresponds to a flow of one ampere for a ten-thousandth of a second. It is evident that such a small amount of electric charge cannot be measured by the method above suggested. Such small quantities of electric charge are measured by means of the *ballistic galvanometer*. This galvanometer is an ordinary D'Arsonval galvanometer. When a momentary pulse of current is sent through such a galvanometer, the suspended coil is set in motion, and a certain *maximum deflection* or *throw* of the coil is produced. Let d be the measure of this maximum deflection or throw on the galvanometer scale. A certain amount of charge Q is represented by the momentary pulse of current, and this amount of charge is proportional to the throw d. That is, we may write:

$$Q = kd \tag{29}$$

in which k is a constant for the given galvanometer, and it is called the *reduction factor* of the galvanometer.

The value of the reduction factor k is generally determined in practice by sending through the galvanometer a known amount of charge Q and observing the throw d produced thereby.

71. Definition of condenser capacity.—*The amount of charge which is drawn out of one plate and forced into the other plate of a condenser is proportional to the electromotive force which acts upon the condenser.* Therefore we may write:

$$Q = CE \tag{30}$$

where Q is the quantity of charge which is drawn out of one plate and forced into the other plate of a condenser when an electromotive force of E volts is connected so as to act upon the condenser, and C is a constant for a given condenser. The factor C is adopted as a measure of what is called the *capacity* of the condenser.

It is evident from the above equation that C, the capacity of a condenser, is expressed in *coulombs-per-volt.* One coulomb-per-volt is called a *farad*, that is to say a *condenser has a capacity of one farad when an electromotive force of one volt will draw one coulomb out of one plate of the condenser and force one coulomb into the other plate of the condenser.*

A condenser has a capacity of one abfarad when one abcoulomb of charge would be drawn out of one plate and forced into the other plate by one abvolt. One abfarad is equal to 10^9 farads.

Condenser capacities as usually encountered in practice are very small fractions of a farad. Thus the capacity of a condenser made by coating with tin foil the inside and outside of an ordinary one-gallon glass jar would be about one five-hundred-millionth of a farad, or 0.002 of a microfarad. A *microfarad* is a millionth of a farad, and in practice capacities of condensers are usually expressed in microfarads.

The approximate dimensions of a one-microfarad condenser are as follows: 501 sheets of tin foil separated by sheets of paraffined paper 0.05 centimeter in thickness, the overlapping portions of the tin foil sheets being 25 centimeters × 25 centimeters, as shown in Fig. 85.

Two pieces of metal of any shape separated by insulating material constitute a condenser; the only reason for using sheets of metal with thin layers of insulating material between is to obtain a large capacity.

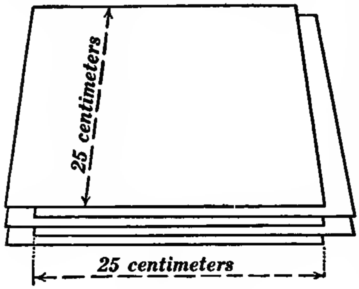

Fig. 85.

Two examples illustrating the use of the ballistic galvanometer.—(*a*) *Comparison of condenser capacities.*—A condenser of which the capacity is C farads is charged by an electromotive force of E volts and discharged through the ballistic galvanometer. Let d be the observed throw of the galvanometer, then

$$CE = kd \tag{i}$$

This equation is evident when we consider that CE is the charge which has been drawn out of one plate of the condenser and forced into the other plate by the charging electromotive force E, and this amount of charge flows through the galvanometer when the condenser is discharged; furthermore the charge which flows through the ballistic galvanometer is equal to kd according to equation (29) of Art. 70.

Another condenser of which the capacity is C' farads is charged by the same electromotive force E, and discharged through the ballistic galvanometer; and the observed throw of the galvanometer is d' scale divisions. Then:

$$C'E = kd' \tag{ii}$$

Dividing equation (i) by equation (ii) member by member, we get

$$\frac{C}{C'} = \frac{d}{d'}$$

A condenser of which the capacity has been carefully measured (at the Bureau of Standards, for example) is called a *standard*

condenser. If C in equation (i) is the known capacity of a standard condenser and if the value of E is known, the value of k (the reduction factor of the galvanometer) may be calculated, the throw d being observed

(*b*) *The measurement of magnetic flux.*—An iron rod has a winding of wire through which a magnetizing current may be passed, and the magnetism of the rod may be reversed by reversing the magnetizing current* this changing the magnetic flux through the rod from $-\Phi$ to $+\Phi$ so that the total change of flux through the rod will be 2Φ. An auxiliary winding of Z turns of wire surrounds the iron rod, this auxiliary winding is connected to a ballistic galvanometer, and d is the observed throw of the ballistic galvanometer when the magnetic flux through the iron rod is suddenly reversed. Then

$$2\Phi = \frac{Rkd}{Z}$$

where R is the total resistance of the ballistic galvanometer circuit (including, of course, the resistance of the auxiliary winding). If R is expressed in ohms, and if k is in coulombs per division, then

$$2\Phi = \frac{Rkd}{Z} \times 10^8 \tag{iii}$$

Derivation of equation (iii). Let $\frac{d\Phi}{dt}$ be the rate of change of the magnetic flux at any instant during the above mentioned reversal. There $\frac{Zd\Phi}{dt}$ is the value of the electromotive force induced in the auxiliary winding (algebraic sign does not concern us) and $\frac{Zd\Phi}{dt} \div R$ is the current in the ballistic galvanometer circuit.† But the current in the ballistic galvanometer circuit is the rate $\frac{dQ}{dt}$ at which electric

* The magnetizing current has to be reversed several times before the reversal of current will change the flux through the rod from $+\Phi$ to $-\Phi$.

† Any delay of current growth due to inductance causes a prolongation of current flow later, and it can be shown that the inductance of the galvanometer circuit has no effect on the total amount of charge which passes through the circuit when Φ is reversed.

charge is passing through the galvanometer. Therefore

$$\frac{dQ}{dt} = \frac{Z}{R} \cdot \frac{d\Phi}{dt}$$

This equation means that Q changes Z/R times as fast as Φ so that total change in Q (total Q passing through the galvanometer) is Z/R times total change in Φ. But the total change of Φ is 2Φ as above explained so that $Q = kd = 2Z\Phi/R$.

PROBLEMS.

93. Imagine the current in a circuit to increase at a constant rate from zero to 50 amperes in 3 seconds. Find the number of coulombs of charge carried through the circuit.

94. An electromotive force which acts on a condenser increases at a constant rate from zero to 1,000 volts during an interval of 0.005 second. The capacity of the condenser is 20 microfarads. Find the value of the current.

95. A 60-cycle alternating electromotive force of which the the maximum value is $\sqrt{2} \times 110$ volts acts on a condenser Find the average value of the current during the 1/120 of a second during which the electromotive force acting on the condenser changes from -155 volts to $+155$ volts, the capacity of the condenser being 20 microfarads.

96. The alternating electromotive force which is represented by the ordinates of the zigzag line in Fig. 86 acts on a condenser

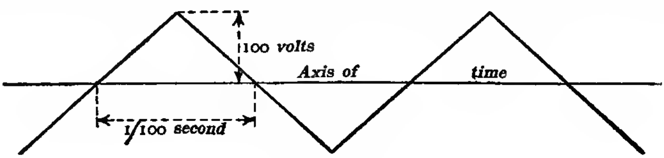

Fig. 86.

of which the capacity is 20 microfarads. Plot a curve of which the ordinates represent the successive instantaneous values of the current flowing into and out of the condenser.

97. An alternating electromotive force $e = E \sin \omega t$ acts on a condenser of which the capacity is C. Find an expression for the current flowing into and out of the condenser.

98. A standard one-microfarad condenser is charged by 14.34 volts and discharged through a ballistic galvanometer producing

a throw of 20.7 scale divisions. What is the value of the reduction factor of the galvanometer in abcoulombs per division?

A very small coil of 100 turns of wire, mean radius of the turns being 1.2 centimeters, is connected to the above ballistic galvanometer, and placed between the poles of a powerful magnet with the plane of the coil at right angles to the lines of force. When the coil is quickly jerked out from between the poles of the magnet the galvanometer throw is observed to be 18.3 divisions. The resistance of the galvanometer circuit (total) is 3,580 ohms. Find the intensity of the magnetic field between the poles of the powerful magnet.

72. **Inductivity of a dielectric.**—The insulating material between the plates of a condenser is called a *dielectric*. Indeed, the insulating material between any two oppositely charged bodies is called a dielectric. The capacity of a condenser depends upon the size of the plates, upon the thickness of the dielectric

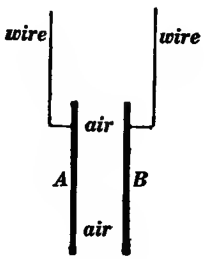

Fig. 87*a*.

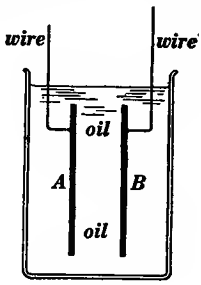

Fig. 87*b*.

and upon the nature of the dielectric. The dependence of the capacity of the condenser upon the nature of the dielectric is a matter which must be determined purely by experiment. Thus Fig. 87*a* represents two metal plates with air between them, and Fig. 87*b* represents the same plates immersed in oil. The distance between the plates is understood to be the same in Figs. 87*a* and 87*b*. Let C be the capacity of the condenser in Fig.

87*a* with air as the dielectric, and let C' be the capacity of the condenser in Fig. 87*b* with a given kind of oil as the dielectric. The ratio C'/C is called the *inductivity** of the oil. Thus the inductivity of kerosene is about 2.04, that is, the capacity of a given condenser is 2.04 times as great with kerosene between the plates as with air between the plates. The accompanying table gives the inductivities of a few dielectrics.

TABLE.

Inductivities of Various Substances.

Crown glass (according to composition)	3.2 to 6.9
Flint glass (according to composition)	6.6 to 9.9
Hard rubber	2.08 to 3.01
Sulphur (amorphous)	3.04 to 3.84
Paraffin	2.00 to 2.32
Shellac	2.74 to 3.67
Ordinary rosin	2.48 to 3.67
Mica (according to composition)	5.66 to 10
Petroleum	about 2.04
Water	about 90.

73. Dependence of capacity of a condenser upon size and distance apart of plates. Consider two flat metal plates with air between. The capacity of this arrangement, as a condenser, is proportional† to a/x, where x is the distance between the plates (thickness of the dielectric) and a is the area of one face of one of the plates (sectional area of dielectric). Therefore the capacity is $C = Ba/x$, or, if the plates are separated by a dielectric of inductivity k, we have:

$$C = B\frac{ka}{x} \tag{31a}$$

The value of B must be determined directly or indirectly by experiment, and its value is

$$B = 884 \times 10^{-16} \text{ farads per centimeter} \tag{31b}$$

The meaning of a may be understood with the help of Fig. 85. If there are 501 sheets of tin foil there will be 500 intervening

* What is here called the *inductivity* of a dielectric is sometimes called *dielectric constant*, or *specific capacity* of a dielectric, or *specific inductive capacity* of a dielectric.

† It can be shown from almost purely geometrical considerations that C is proportional to a/x, but it is sufficient to accept this proportional relationship as the result of experiment.

leaves of dielectric (waxed paper, for example) and a will be $500 \times 25 \times 25$ square centimeters.

Maxwell's theory shows that $B = 10^9/(4\pi v^2)$ where v ($= 2.998 \times 10^{10}$ centimeters per second) is the velocity of light in air.*

PROBLEMS.

99. Two parallel metal plates at a fixed distance apart with air between are charged as a condenser, and discharged through a ballistic galvanometer. The plates are then submerged in turpentine and again charged and discharged though the same ballistic galvanometer. The charging electromotive force is the same in each case, and the throw of the ballistic galvanometer is observed to be 7.6 divisions in the first instance and 16.7 divisions in the second instance. Find the inductivity of the turpentine.

100. A condenser is to be built up of sheets of tin foil 12 centimeters by 15 centimeters. The overlapping portions of the sheets are to be 12 centimeters by 12 centimeters. The sheets are to be separated by leaves of mica 0.05 centimeter thick. How many mica leaves and how many tin foil sheets are required for a one-microfarad condenser? Assume the inductivity of the mica to be equal to 6.

101. A condenser is made of two flat metal plates separated by air. Its capacity is 0.003 microfarad. Another condenser has plates twice as wide and twice as long. These plates are separated by a plate of glass (inductivity 5) which is four times as thick as the air space in the first condenser. What is the capacity of the second condenser?

102. Prove that the capacity of two or more condensers connected in parallel is equal to the sum of the capacities of the individual condensers.

103a. A 20-microfarad condenser and a 10-microfarad condenser are connected in series. Suppose that 5 microcoulombs

* A very simple discussion of this matter is given on pages 240–253 of the first edition (1896) of Nichols and Franklin's *Elements of Physics*, Volume II.

of charge is "pulled through" the two condensers by an electromotive force E. Find the voltage across each condenser, calculate the value of E, and find the capacity of a single condenser through which E would "pull" 5 microcoulombs.

103b. Prove that the combined capacity of a number of condensers connected in series is equal to the reciprocal of the sum of the reciprocals of the capacities of the individual condensers.

Note. Let q be the charge which is "pulled through" the set of condensers by E volts. Then E volts would, by definition, "pull" q coulombs "through" a single condenser whose capacity is equal to the combined capacity of the set.

103c. Calculate the combined capacity of three condensers in series whose individual capacities are 5 microfarads, 10 microfarads and 15 microfarads.

---

74. The work done by an electromotive force E in pushing a given amount of charge, Q, through a circuit.—When Q coulombs of electric charge flow through a battery of which the electromotive force is E, the amount of work W done by the battery is EQ joules. That is:

$$W = EQ \tag{32}$$

This is evident from the following considerations. Imagine a current I flowing through the battery; then EI watts is the rate at which the battery does work, and EIt joules is the amount of work done in t seconds. But the product It is the amount of charge Q (in coulombs) which has been pushed through the circuit. Therefore the work done, namely EIt joules, is expressible as EQ joules.

75. The potential energy of a charged condenser.—A charged condenser represents a store of potential energy in much the same way that a stretched spring or the distorted diaphragm DD in Fig. 83*a* represents a store of potential energy.

A condenser is charged by applying it to an electro-motive force which begins at zero and rises to E volts, and the amount of work W which is done in charging the condenser is equal to

$\frac{1}{2}EQ$, where $\frac{1}{2}E$ is the average value of the charging electromotive force, and Q is the total charge which is drawn out of one plate of the condenser and pushed into the other plate. This statement is in accordance with equation (32) of Art. 74. Therefore:

$$W = \tfrac{1}{2}EQ \tag{33}$$

where W is the potential energy of a charged condenser, E is the voltage acting on the charged condenser, and Q is the charge which has been drawn out of one plate of the condenser and pushed into the other plate; W is expressed in joules when E is in volts and Q in coulombs.

We may substitute CE for Q in equation (33), according to equation (30) of Art. 71, and we get:

$$W = \tfrac{1}{2}CE^2 \tag{34}$$

or we may substitute Q/C for E in equation (33), according to equation (30) of Art. 71, and we get:

$$W = \tfrac{1}{2}\frac{Q^2}{C} \tag{35}$$

Following is a rigorous derivation of equation (35) as applied to a stretched spring. Let q be the elongation of the spring when the stretching force is e. Then q and e are proportional, so that:

$$q = Ce \tag{i}$$

where C is a constant for the given spring. Let Δq be the added elongation due to an increment Δe of the stretching force, and let ΔW be the work done on the spring to produce the added elongation. Then:

$$\Delta W \text{ is greater than } e\cdot\Delta q$$

and

$$\Delta W \text{ is less than } (e + \Delta e)\cdot\Delta q$$

or

$$\frac{\Delta W}{\Delta q} \text{ is greater than } e \text{ and less than } (e + \Delta e).$$

Therefore $\Delta W/\Delta q$ approaches e as a limit when Δe and Δq both approach zero or, using differential notation, we have $dW/dq = e$; or, using the value of e from equation (i), we have:

$$\frac{dW}{dq} = \frac{q}{C} \tag{ii}$$

Now the potential energy W of the spring when its elongation is Q, is the

amount of work done in stretching the spring from $q = 0$ to $q = Q$, and this is found by integrating equation (ii) from $q = 0$ to $q = Q$, which gives:

$$W = \frac{1}{2}\frac{Q^2}{C} \tag{iii}$$

PROBLEMS.

104. Given two insulated, flat, metal plates 2 meters × 2 meters with a two-centimeter air space between. Find the capacity of the two plates as a condenser. Find the energy of this condenser in joules when it is charged from 100-volt, direct-current, supply mains. What would the energy be if the charging electromotive force were 10,000 volts?

105. The condenser which is specified in the previous problem is charged by an electromotive force of 10,000 volts, *the plates are then insulated so that Q cannot change*, and the plates are then drawn apart so that the air space is 4 centimeters. What is the electromotive force between the plates after they are drawn apart? How much is the energy of the condenser increased by pulling the plates apart? How much work has to be done to pull the plates apart? How much force (the force is in fact constant so that we do not have to say average force) is required to pull the plates apart? What is the electrostatic attraction of the plates?

Note.—The unit of force in the ampere-volt-ohm system may be called the *joule per centimeter* inasmuch as it is the force which will do one joule of work when the body on which it acts moves one centimeter in its direction.

106. Find each result asked for in the previous problem, the plates being charged by 100 volts when at a distance of 2 centimeters apart, and then insulated so as to keep Q from changing.

107. Find each result asked for in problem 105 if the plates are all the time immersed in petroleum, charged by 10,000 volts when they are 2 centimeters apart, and then insulated (if this were possible) so as to keep Q constant.

108. Substitute the value of C from equation (31*b*) in equation (35), assume the parallel plates to be perfectly insulated so that Q cannot change, differentiate with respect to x, and find

the general expression for the force F with which parallel plates attract each other.

Note.—The force F is equal to $\frac{dW}{dx}$. Why?

---

76. Disruptive discharge. Dielectric strength.—When the electromotive force which charges a condenser is increased more and more, the dielectric of the condenser is eventually broken down; this break-down occurs in the form of an electric spark, it discharges the condenser, and it is called a *disruptive discharge.* By a condenser is here meant two metal bodies of any shape separated by insulating material. The electromotive force required to break down a dielectric depends upon three things, namely, (*a*) the shape of the metal bodies, (*b*) the minimum distance* between the metal bodies, and (*c*) the nature of the dielectric. The dependence upon the shape of the metal bodies is illustrated by the fact that a given electromotive force will produce a much longer spark between points than between flat metal surfaces. *In the whole of the following discussion the dielectric is assumed to be between flat metal plates.*

When the dielectric is perfectly homogeneous like air or oil, the voltage required to break it down is very nearly proportional to its thickness, and the voltage required to break down such a dielectric divided by the thickness of the dielectric is called the *specific strength* of the dielectric. Thus the specific dielectric strength of air is about 35,000 volts per centimeter. When the dielectric is non-homogeneous the voltage required to break it down is not even approximately proportional to its thickness. The most familiar example of a non-homogeneous dielectric is the material which is used for insulating the windings of dynamos and transformers. This material is made up of layers of cloth and varnish and mica with occasional layers of air.

If a tank is made with one wall of porous material like unglazed earthenware, the pressure of the fluid in the tank has three

* This is not true when the distance is very small or when the bodies are in a very good vacuum.

important effects upon the wall, namely, (*a*) fluid soaks through the wall at a certain rate, (*b*) the wall is slightly elastic and it yields a little to the fluid pressure, and (*c*) the wall has a certain ultimate strength and it will burst if the pressure exceeds a certain amount. Similarly the electromotive force which acts on a condenser has three important effects upon the dielectric of the condenser, namely, (*a*) a certain amount of electric current "soaks" through the dielectric as it were, because the dielectric is an electrical conductor although a very poor one, (*b*) the dielectric has a certain amount of electrical "elasticity" (inductivity as it is properly called), and it "yields" a little to the electromotive force and allows a certain amount of charge to be drawn out of one plate and forced into the other plate of the condenser, and (*c*) the dielectric has a certain ultimate strength and it will be ruptured if the electromotive force exceeds a certain amount.

77. **The disruptive discharge as a means for exciting electric oscillations.**—A condenser *C*, Fig. 88 (usually consisting of a

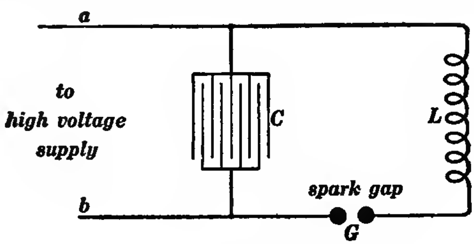

Fig. 88.

number of glass jars with coatings of tinfoil outside and inside, called *Leyden jars*), is connected to the high-voltage terminals *a* and *b* of a step-up transformer. As the voltage between *a* and *b* rises (alternating voltage, of course) the condenser becomes charged; eventually the spark gap *G* breaks down, and then the charge on the condenser surges back and forth through the inductance coil *L* and across the air gap *G* until the energy of the charged condenser is dissipated. When the back and

forth surges cease, the air gap G quickly cools* and regains its insulating power, the condenser is again charged until the air gap again breaks down when another series of back and forth surges takes place, and so on.

The condenser C and the inductance L connected as shown in Fig. 88 constitute an *electric oscillator.*

The Hertz oscillator.—Two brass rods A and B, Fig. 89, have a spark gap G between them. The rods are connected to the high-voltage terminals of an induction coil, and at each interruption of the primary circuit of the induction coil the rods A and B are charged sufficiently to produce a spark across the air gap G. This spark is a sudden breakdown of the insulation of the gap, and this break-down is followed by a back and forth surging of current across the gap and along the rods A and B.

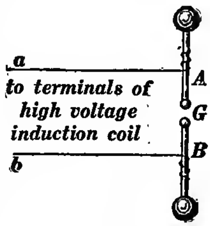

Fig. 89.

The type of electric oscillator shown in Fig. 89 was devised by Heinrich Hertz in 1888, and this type of oscillator gives off a large portion of its energy in the form of electric waves.

The use of the electric oscillator in wireless telegraphy.—Figure 90*a* shows the essential features of a simple type of sending station for wireless telegraphy, and Fig. 90*b* shows the essential features of a receiving station.† The arrangement *abCLG* in Fig. 90*a* is the electric oscillator which is shown in Fig. 88. The two coils L and S constitute a transformer. The back and forth surging of the oscillating current in coil L induces a high-frequency electromotive force in the coil S, this electromotive force causes high-frequency current to surge up and down in the antenna, and electric waves pass out from the antenna. These electric waves produce up and down surges of current

* The air in the path of an electric spark owes its electrical conductivity not only to high temperature, but also, and indeed chiefly, to the fact that the air molecules are broken up into charged atoms which are called ions.

† The arrangement for "tuning" is not shown in Fig. 90*b*.

(very weak) in the antenna at the receiving station, these up and down surges of current flow through the *primary coil* S of a transformer in Fig. 90*b*, and every alternate half-wave of the

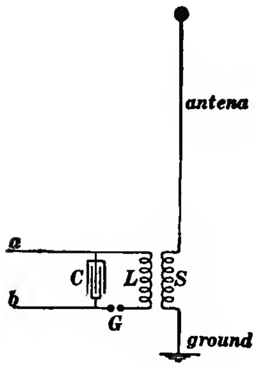

Fig. 90*a*.

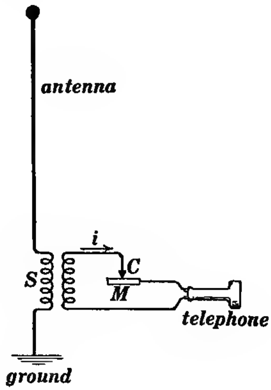

Fig. 90*b*.

secondary current i flows through the crystal rectifier CM and through a telephone receiver.

The antenna as usually constructed consists of a horizontal band of parallel wires stretched between two cross-pieces and supported by two masts, as shown in Fig. 91. The crystal

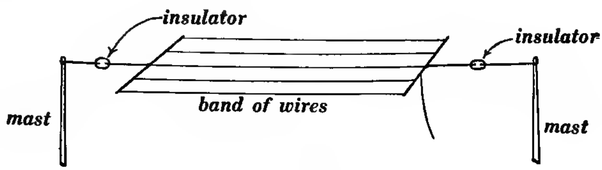

Fig. 91.

rectifier consists of a fragment of a crystal of galena C resting lightly on a metal plate M.*

78. The frequency equation of the electric oscillator.—The most convincing derivation of the frequency equation of the electric oscillator is to point out the identity of this equation to

* The crystal rectifier has been largely superseded by the audion as explained in Chapter VII.

the equation in mechanics which expresses the frequency of the oscillations of a weight attached to a spring as follows:

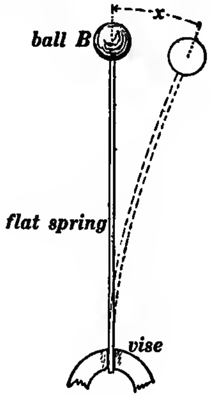

Fig. 92.

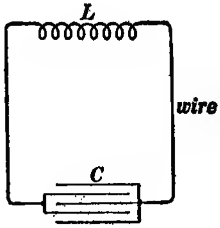

Fig. 93.

When the ball is at a distance x from its equilibrium position, the spring reacts on the ball and exerts on the ball an unbalanced force F which is given by the equation

$$F = -kx \qquad \text{(i)}$$

where k is what is sometimes called the "stiffness coefficient" of the spring.

If an amount of charge q is drawn out of one plate and forced into the other plate of the condenser thus charging the condenser, the charged condenser reacts on the circuit and exerts on the circuit an electromotive force E which is given by the equation

$$E = -\frac{1}{C}\cdot q \qquad \text{(ii)}$$

This is in accordance with equation (30), the negative sign being introduced because we are considering *not* the electromotive force which is acting on the condenser but the equal and opposite electromotive force with which the condenser is reacting on the circuit.

If no friction exists the force F is an unbalanced force acting on the ball and therefore

$$F = m\frac{dv}{dt} \qquad \text{(iii)}$$

where m is the mass of the ball, $v\left(=\frac{dx}{dt}\right)$ is the velocity of the ball, and $\frac{dv}{dt}$ is the rate of increase of the velocity of the ball (the acceleration of the ball).

If the circuit in Fig. 93 has no resistance then all of the electromotive force E is used to cause the current i in the circuit to increase in accordance with equation (20) of Art. 65. Therefore

$$E = L\frac{di}{dt} \qquad \text{(iv)}$$

where L is the inductance of the circuit, $i\left(=\frac{dq}{dt}\right)$ is the current flowing at any instant, and $\frac{di}{dt}$ is the rate at which the current is increasing.

If the ball in Fig. 92 is pulled to one side and released it will perform harmonic oscillatory motion such that

$$4\pi^2n^2m = k \qquad \text{(v)}$$

where n is the number of complete oscillations per second.*

If the condenser in Fig. 93 is charged and the circuit closed, a current will surge back and forth through the circuit, and we will have

$$4\pi^2n^2L = \frac{1}{C} \qquad \text{(vi)}$$

where n is the number of complete oscillations per second.†

Equation (vi) is usually written in the form

$$n = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \qquad (36)$$

PROBLEM.

109a. Find the potential energy of a 10-microfarad condenser when charged by an electromotive force of 1000 volts. Express the result in joules and in ergs. How much potential energy is left in the condenser when it has lost half its charge?

* See Franklin and MacNutt's *Lessons in Mechanics*, Art. 46.

† This matter is discussed more at length in Appendix B.

109b. A 10-microfarad condenser is charged by an electromotive force of 1000 volts, and its terminals are then disconnected from the charging electromotive force and connected to the terminals of an uncharged 20-microfarad condenser. Find the potential energy of the two condensers.

109c. How does the force of electrostatic attraction of two large flat metal plates, with air or oil between, vary with the distance between the plates and with the inductivity of the dielectric (*a*) for a given amount of charge $\pm Q$ on the plates, and (*b*) with a given electromotive force E between the plates?

Note. The formula found in problem 108 will answer (*a*) by inspection. To answer (*b*) substitute C from equation (31*a*) on page 111 in equation (30) on page 105, and substitute the value of Q so found in the equation derived in problem 108.

109d. A condenser consisting of two metal plates 100 centimeters square with one millimeter of petroleum between is connected to a coil consisting of 200 turns of wire wound on a paper mailing tube 5 centimeters in diameter and 30 centimeters long. Calculate the frequency of the oscillations when the charged condenser is suddenly connected to the terminals of the winding.

Note.—Equation (36) expresses the frequency when the resistance of the circuit is negligibly small; and as a matter of fact the frequency is but little affected by the resistance unless the resistance is quite large. Similarly the resistance or friction which opposes the motion of a pendulum has but little influence on the frequency of the oscillations of the pendulum.

The correct expression for n is given in Appendix *B*.

A very helpful exhibit of the equations of translatory motion, of the simpler equations of rotatory motion, and of the simpler equations of "electrical motion" is given in Appendix D.

109e. A condenser of capacity C discharges through a very high non-inductive resistance R. Set up the differential equation of the decreasing voltage across the condenser.

CHAPTER VI.

THE ELECTRIC FIELD. ELEMENTARY THEORY OF POTENTIAL.

79. The electric field.—When a momentary electric current flows through an open circuit, certain important effects are produced in the gap which breaks the circuit. In order that these effects may be easily observed a very high voltage must be used. The most convenient device for generating a high voltage is the influence electric machine which is described in Art. 89. Figure 94 shows two brass balls which have been charged by a momentary electric current drawn out of one ball and pushed into the other by an influence machine. When an ordinary wooden toothpick, suspended by a fine thread, is placed in the region between the balls, the toothpick points in a definite direction at each point very much as a magnetic needle points in a definite direction at each point when it is placed between magnet poles. The short black lines in Figs. 94 and 95 represent the various positions of the toothpick.

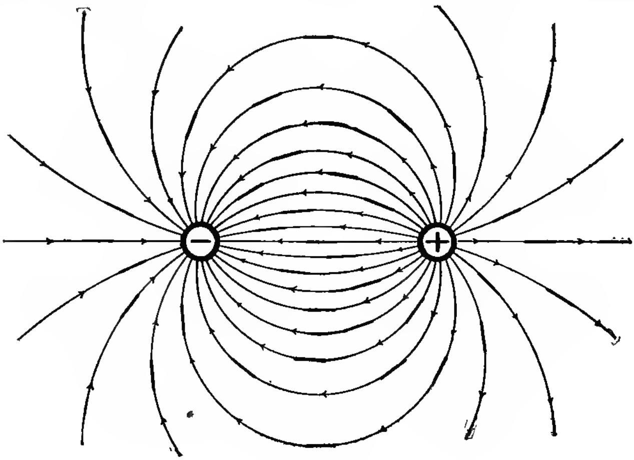

Fig. 94.

The behavior of the toothpick shows that the whole region surrounding the charged metal balls in Fig. 94 is in a peculiar condition, and this region is called an *electric field*. The direction of the electric field at each point is indicated by the direction of the toothpick when it is placed at that point, and lines drawn through the electric field so as to be, at each point, in the direction of the field at that point are called the *lines of force* of the electric field. Figure 95 shows the lines of force

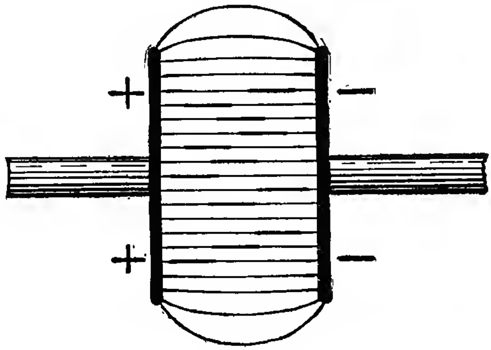

Fig. 95.

of the electric field between two charged flat metal plates. The lines of force in the region between the plates are straight lines, and the electric field is said to be *uniform*.

80. Intensity of electric field.—It would be permissible to adopt arbitrarily the ratio E/x as a measure of the intensity of the uniform electric field between the flat metal plates in Fig. 95, E being the electromotive force between the plates and x being the distance between the plates. Thus the intensity of the electric field would be expressed in *volts per centimeter* or *volts per inch*. It is desirable, however, to base the definition of electric field intensity upon some observable effect as in the following discussion.

Two metal plates A and B, Fig. 96, are connected to an electric machine giving a high electromotive force E. The electric machine is represented in Fig. 96 as a battery for the

sake of clearness. A small metal ball b is suspended between A and B by a silk thread. If this ball is started it continues to vibrate back and forth from plate to plate.

Regarding the behavior of the vibrating ball the following statements may be made:

(*a*) Work evidently is done to keep the ball b oscillating back and forth, and this work is evidently done by the battery.

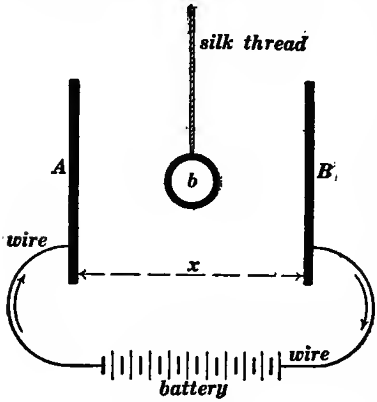

Fig. 96.
The ball b oscillates to and fro.

(*b*) The only way the battery can do work is by continuing to draw charge out of one plate and push it into the other plate. It is evident therefore that the ball carries charge back and forth between the plates.

(*c*) The successive movements of the ball are similar, and therefore if the ball carries charge at all it must carry a definite amount each time it moves across. Let this definite amount of charge be represented by q; this charge is positive when the ball moves from A to B, and negative when it moves from B to A. At each movement of the ball the battery supplies the amount of charge q, drawing it out of plate B and pushing it into plate A. Therefore at each movement of the ball the battery does an amount of work Eq according to equation (32) of Art. 74.

(*d*) Let F be the average mechanical force acting on the ball b while it is being pulled across from plate to plate. Assuming the ball b to be very small in diameter, it moves the distance x in traveling from plate to plate. Then Fx is the amount of work done on the ball while it moves from plate to plate.

(*e*) The work Eq done by the battery during one movement of the ball is equal to the mechanical work Fx done on the ball,

therefore we have $Fx = Eq$, or

$$F = \frac{E}{x} \cdot q \qquad \text{(i)}$$

Any region in which a charged body is acted upon by a force* is called an electric field. Thus the region between A and B in Fig. 96 is an electric field because the charged ball b is acted upon by the force F.

The force F with which an electric field pulls on a charged body placed at a given point in the field is proportional to the charge q on the body so that we may write:

$$F = fq \qquad (37)$$

in which f is the proportionality factor, and it is called the *intensity of the electric field at the point.*

From equations (i) and (37) it is evident that the intensity of the electric field between the plates A and B in Fig. 96 is:

$$f = \frac{E}{x} \qquad (38)$$

that is, the intensity of the electric field between the plates is equal to the electromotive force between the plates divided by the distance between the plates. In the above discussion F is spoken of as the average force acting on the ball b in Fig. 96 while the ball is moving from plate A to plate B. As a matter of fact this force is constant if the ball b is very small.

Direction of electric field at a point.—The direction of an electric field at a point is the direction in which the field would pull on a *positively charged body* placed at that point.

PROBLEM.

110. A very small ball weighing 0.1 gram is suspended by a short silk fiber between the two plates A and B in Fig. 96, and after the ball touches one of the plates the suspending fiber stands 20° from the vertical. The distance between A and B

* A force which depends upon the charge on the body and which does not exist when the body is not charged.

is 25 centimeters and the electromotive force between A and B is 50,000 volts. What amount of charge is there on the ball? Ans. 1.78×10^{-9} coulombs.

81. The idea of electric charge as the ending of electric lines of force.—Figure 97 represents two metal bodies A and B to which a battery is connected as shown. The battery draws a certain amount of charge out of one body B and forces it into the other body A, and the entire surrounding region becomes an electric field, the lines of force of which are shown in the figure. *The positive charge on body A may be thought of as the beginning of the lines of force, and the negative charge on body B may be thought of as the ending of the lines of force,* the directions of the lines of force being indicated by the arrow heads in the figure.

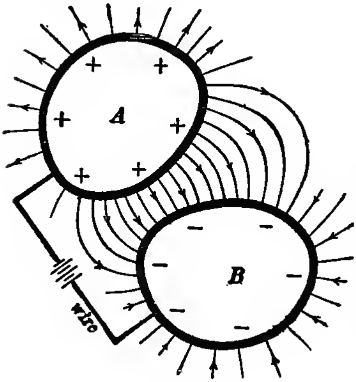

Fig. 97.

82. The pith-ball electroscope.—The presence of an electrical field may be shown by the behavior of a suspended wooden toothpick as described in Art. 79, and such a device may therefore

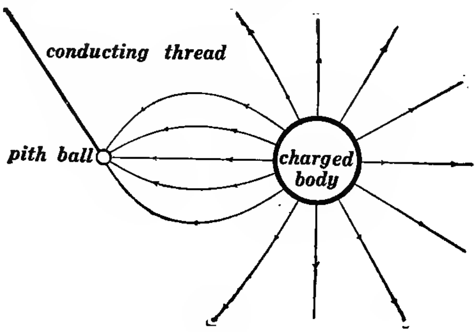

Fig. 98.

be called *an electroscope*. A more sensitive electroscope is made by suspending a small pith ball by a very fine slightly conducting thread. When a charged body is brought near to such a suspended pith ball the ball becomes charged as indicated by the lines of force in Fig. 98, and the lines of force from the charged body to the ball pull the ball towards the body as shown. The suspending thread may be made slightly conducting by soaking it in a dilute salt solution and allowing it to dry.

83. Electric charge resides wholly in the surface of a metal body.—Experiment shows that to whatever degree a hollow metal shell may be charged, no effect of the charge can be observed inside of the shell, however thin the shell may be; that is to say, the lines of force of the outside electric field do not penetrate into the metal but terminate at its surface. Therefore, the electric charge on a metal body may be thought of as residing on the surface of the body. Figure 99 shows a hollow metal ball C placed between two charged bodies A and B.

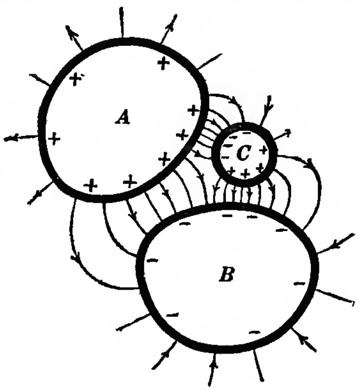

Fig. 99.

The presence of the ball C modifies the trend of the lines of force as may be seen by comparing Fig. 99 with Fig. 97, but the lines of force do not penetrate to the interior of the ball C. *The interior of a metal shell is entirely screened from outside electric field.** This is an experimental fact. An electric field may be detected by its action upon a very light body like a suspended toothpick or a suspended pith ball. No evidence of

Fig. 100.

* The screening is not complete while an electric field is changing rapidly.

electrical field can be detected inside of the ball C by such a device.

Mechanical analogue of electrical screening.—Consider a mass of steel B, Fig. 100, which is entirely separated from a surrounding mass of steel by an empty space *eee*. Stress and distortion of the surrounding steel cannot affect B in any way, and conversely stress and distortion of B cannot affect the surrounding steel, because the empty space is incapable of transmitting stress. This empty space, in its behavior towards mechanical stress, is analogous to a metal (or any electrical conductor) in its behavior towards electrical stress (electrical field).

84. A charged conductor shares its charge with another conductor with which it is brought into contact.—A brass ball with a glass handle may be charged by touching it to one terminal of an influence machine, and if the brass ball is brought near to a suspended pith ball, as shown in Fig. 98, the charge on the brass ball will be indicated by the behavior of the pith ball.

A brass ball A is charged by touching it to a terminal of an

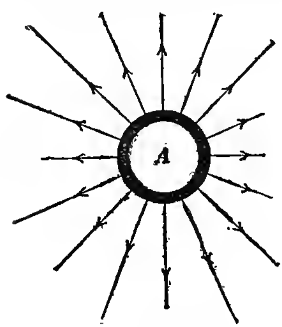

Fig. 101.
Charge on single ball.

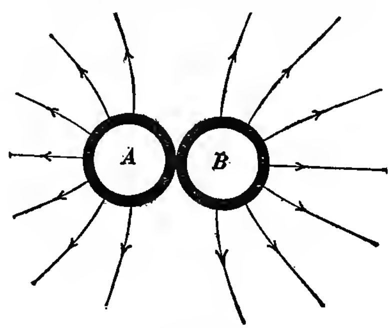

Fig. 102.
Charge shared by two balls.

influence machine, *the ball* A *is then touched to another brass ball* B (A and B both have glass handles), *then both* A *and* B *are found to be charged.* The charged ball A has shared its charge with ball B. The original charge on ball A is represented in Fig.

101, which shows the lines of force emanating from ball A; and the lines of force in Fig. 102 show how the charge originally on A has spread over A and B.

85. Giving up of entire charge by one body to another.—Figure 103 shows a charged ball and an insulated metal can. If

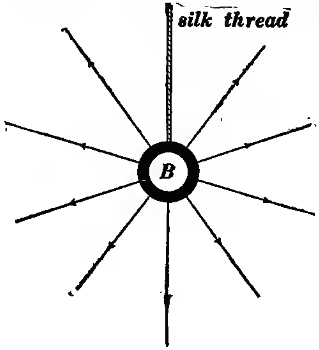

Fig. 103*a*.

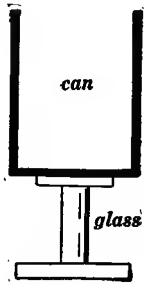

Fig. 103*b*

the charged ball is placed inside of the can, brought into contact with the inner wall of the can, and then removed, it will give up its entire charge to the can. *This is true whatever charge the can may have to begin with.* This giving up of the entire charge

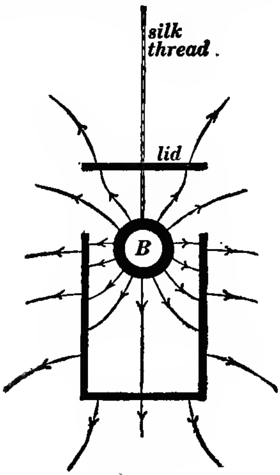

Fig. 104.

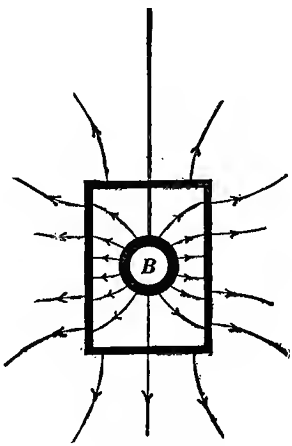

Fig. 105.

on one body to another, as described, is an essential feature in the action of the electric doubler and of the influence machine (see Arts. 87 and 89).

The charged ball B is lowered into the can, and the can is closed by the metal lid as indicated in Figs. 104 and 105. As the charged ball is lowered into the can each line of force that emanates from the ball is cut in two, as it were, by the wall of the

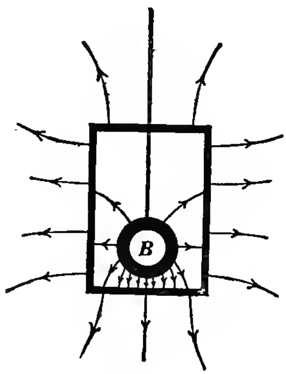

Fig. 106.

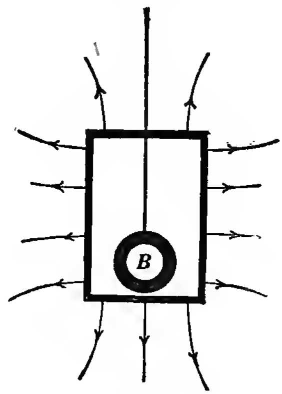

Fig. 107.

can so that when the ball is entirely enclosed by the can as many lines of force emanate from the external surface of the can as from the ball B, as shown in Fig. 105.

After the ball B has been completely inclosed by the metal can, the distribution of the electric field outside of the can is entirely independent of what takes place inside of the can, because the walls of the can act as a complete screen as explained in Art. 83.

If the ball B is then brought into contact with the inner wall of the can, as shown in Figs. 106 and 107, the lines of force which emanate from the ball disappear, as shown in Fig. 107, and all of the charge originally on B is left on the outside surface of the can. The ball may then be removed from the can and it will be found to be without charge, all of its original charge will have been given to the can. This giving up of the entire charge

by the ball takes place however great the initial charge on the can may be.

86. Charging by influence.—Charging by influence is essentially the cutting of electric lines of force in two by a sheet of metal so that one face of the metal sheet is negatively charged where the lines of force come in upon it, and the other face of

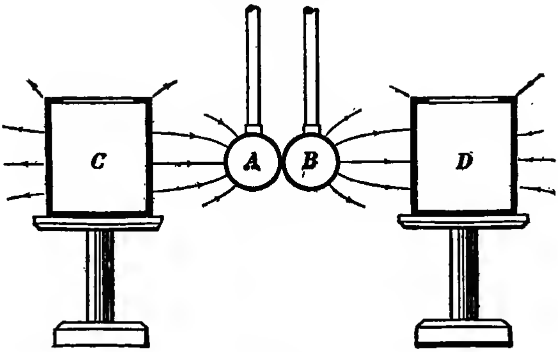

Fig. 108.

the metal sheet is positively charged where the lines of force emanate from it. Thus Fig. 108 shows two metal balls *A* and *B* which have been brought in between two oppositely charged bodies *C* and *D*. The lines of force from *C* converge upon *A*,

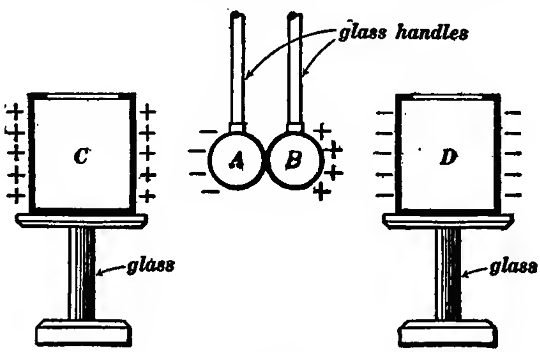

Fig. 109.

and spread out from *B* as shown. If *A* and *B* are now separated from each other and withdrawn from the region between

C and D, then B will be left positively charged (lines of force emanating from it), and A will be left negatively charged (lines of force coming in upon it); and the charges on C and D will be the same as at the beginning.

87. **The electric doubler.**—The charged bodies C and D in Fig. 108 are metal cans supported on insulating stands as shown also in Fig. 109. The ball A may be made to give up its entire charge to D by being placed inside of D and brought into contact with D; and the ball B may be made to give up its entire charge to C in a similar manner. The two balls A and B may then be again charged by being brought into the positions shown in Figs. 108 and 109, and the charges on A and B may again be delivered to D and C as before; and so on. In this way the two cans C and D may be charged to any degree whatever, starting with any initial charges however small, pro-

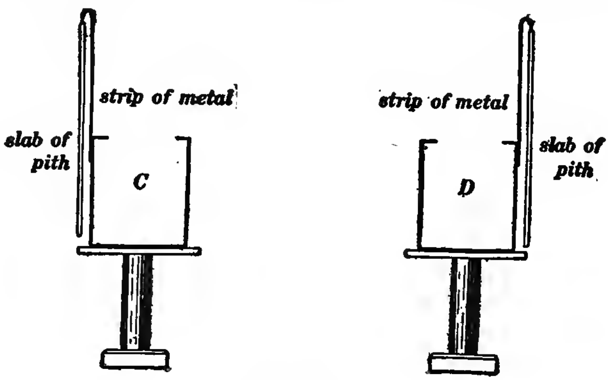

Fig. 110.

vided the insulation is very good. If the insulation is poor the charges leak away rapidly.

A very interesting and striking experiment is the following. Two thin slabs of pith are attached to the cans C and D as shown in Fig. 110. Then fifty or more repetitions of the above described operation will charge both C and D sufficiently to make the thin slabs of pith stand out nearly horizontally. The success of this experiment depends upon extremely good insula-

tion. The cans C and D should be supported upon freshly scraped blocks of hard paraffine or of cast sulphur, and the handles of the balls A and B should be fused quartz tubes closed at one end.

88. The gold leaf electroscope.—The essential features of the gold leaf electroscope are shown in Fig. 111. A metal rod R is supported in the top of a glass case cc by means of an insulating plug. This plug is preferably made of cast sulphur. A metal disk D is fixed to the upper end of the rod, and two strips of gold leaf are hung side by side from the lower end of the rod. The

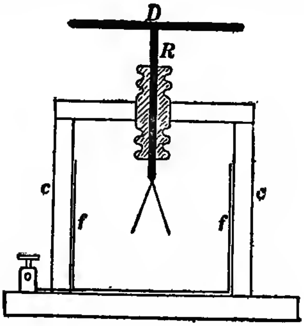

Fig. 111.
The gold-leaf electroscope.

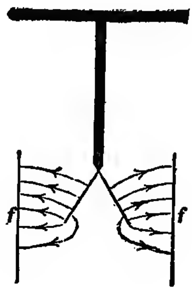

Fig. 112.

glass case cc serves to protect the gold leaves from draughts of air. The sides of cc should be lined with metal strips ff, and these strips should be connected to earth. When the disk, rod and leaves are charged, the leaves are pulled apart by the lines of force which emanate from the leaves and terminate on the strips ff, as shown in Fig. 112.

The behavior of a gold leaf electroscope is as follows: (1) When the electroscope has no initial charge, the gold leaves diverge when a positively or negatively charged body is brought near to the disk D. (2) If the disk or rod is touched with the finger when, say, a positively charged body is near D, then the disk and leaves are left with a negative charge when the positively charged body is removed to a distance. This is the operation of

charging by influence which is described in a general way in Art. 86. (3) When the disk and leaves have an initial charge, the divergence of the leaves is increased by bringing a body with a like charge near D, and the divergence of the leaves is decreased by bringing a body with an unlike charge near D.

89. The Toepler-Holtz influence machine.—The action of the Toepler-Holtz machine is essentially like the action of the electric doubler as described in Art. 87, except that in the Toepler-

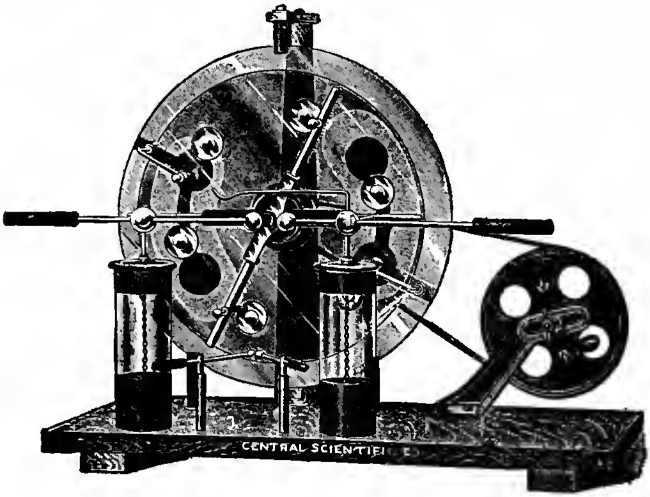

Fig. 113.
The Toepler-Holtz Electric Machine.

Holtz machine each can in Fig. 108 *is made of two separate parts.* Thus C and C' in Fig. 114 together take the place of C in Fig. 108, and D and D' together take the place of D in Fig. 108.

A general view of a Toepler-Holtz machine is shown in Fig. 113, and the essential features of the machine are shown in Fig. 114. Metal carriers *ccc* travel along the dotted line in the direction indicated by the arrows. In positions 1 and 4 these carriers are under the influence of the charged bodies C and D, and they touch the neutralizing rod so that one carrier is left with an excess of negative charge and the other carrier is left with an

excess of positive charge as shown. When the carriers reach the positions 2 and 5 they are momentarily connected to the charged bodies or *inductors* C and D to which they give up a portion of their charges. The inductors C and D are thus kept charged. When the carriers reach the positions 3 and 6 they make momen-

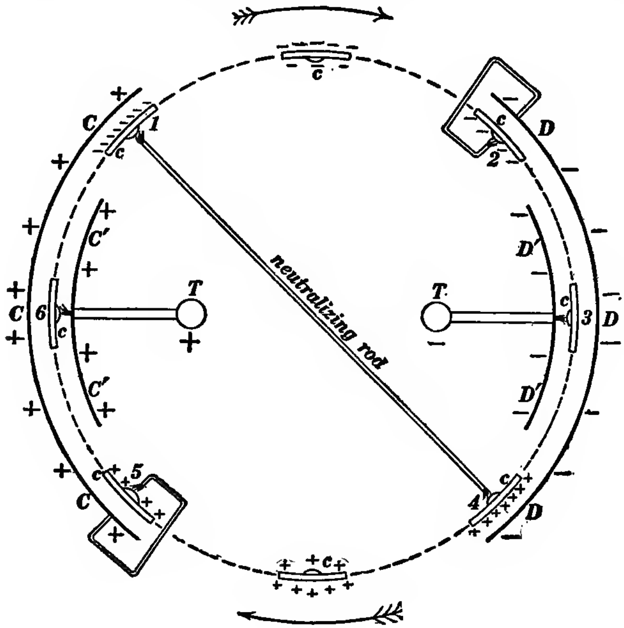

Fig. 114.

tary contact with C' and D' to which they give up nearly all of their charges. This action is repeated over and over again.

When the two cans in Fig. 108 are discharged it takes a long time for the doubling action to bring them up to a highly charged condition again. This difficulty is obviated in the Toepler-Holtz machine, because the terminals TT of the machine are not connected to the inductors C and D, and therefore the inductors C and D are not discharged when the terminals TT are connected together, but they continue to exert a strong charging influence upon the metal carriers as they pass positions 1 and 4.

90. Electric flux. Gauss's theorem.—Consider an area of a square centimeters at right angles to an electric field of which the intensity is f volts per centimeter. The product af is called the *electric flux* across the area, and it is, of course, expressed in *volt-centimeters* or *abvolt-centimeters.*

The conception of electric charge as the beginning or ending of lines of force of the electric field is very inadequately explained in Art. 81, and this conception when fully apprehended is intimately connected with a theorem which is due to Gauss, namely, *the total electric flux which emanates from a positively charged body is*

$$\Psi = \frac{q}{B} \tag{i}$$

where Ψ is the electric flux in volt-centimeters, q is the total charge on the body, and B is a constant, namely,

$$B = 884 \times 10^{-16} \tag{ii}$$

When the charge is negative the flux comes in towards the body.

Application of Gauss's theorem to two parallel and oppositely charged metal plates.—Let us consider the uniform electric field between two flat metal plates as indicated in Fig. 95. The electric field is thought of as being directed away from the positively charged plate and towards the negatively charged plate, and the amount of flux crossing from plate to plate is $a \times \frac{E}{x}$; where a is the area of one of the plates, E is the electromotive force between the plates and x is the distance of the plates apart. Now the condenser capacity of the two plates is given by equation (31*a*) of Art. 73 so that the charge q on either plate (positive and negative on the respective plates) is

$$q = CE = 884 \times 10^{-16} \times \frac{a}{x} \cdot E$$

so that

$$q = Ba\frac{E}{x}$$

But E/x is the intensity of the electric field between the plates, so that that aE/x is the flux Ψ from plate to plate, and therefore we have $q = B\Psi$, which is equation (i).

Let us consider two parallel metal plates with oil between, thickness of oil layer being x centimeters and inductivity of oil being k. The condenser capacity of the two plates is given by equation (31*b*) of Art. 73 so that in this case we have

$$q = CE = Bakf$$

where f is written for E/x. In this case akf is the electric flux from plate to plate and kf is the electric flux per unit area or the *electric flux density* as it is called.

Electric flux density in air is equal to electric field intensity.

Electric flux density in oil or any other dielectric is equal to the product of the electric field intensity and the inductivity k of the dielectric.

91. Continuity of electric flux or of electric flux density. Electric stresses in a stratified dielectric.—Figure 115 shows a layer of glass of inductivity k_g and a layer of oil of inductivity k_0 between two flat metal plates. The electromotive force between the plates is E volts, thicknesses of glass and oil layers are x centimeters any y centimeters, respectively, as shown, and it is desired to find the electric field intensity or stress in volts per centimeter in each of the dielectrics, glass and oil.

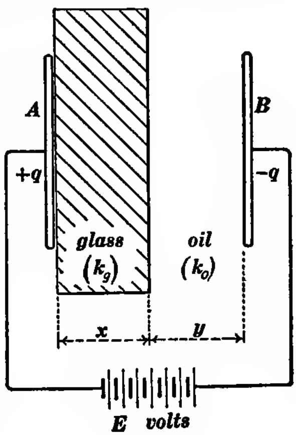

Fig. 115.

According to Gauss's theorem a certain amount of electric flux crosses from the positively charged plate to the negatively charged plate, and it is evident that the flux density is the same in glass and in oil. Therefore we have

$$k_0 f_0 = k_g f_g \tag{i}$$

where f_0 is the electric field intensity in the oil and f_g is the electric field intensity in the glass.

Now f_g being the volts per centimeter in the glass, xf_g is of course, the total volts across the glass layer; and similarly, yf_0 is the total volts across the oil layer. Therefore we have

$$E = xf_g + yf_0 \qquad \text{(ii)}$$

and from equations (i) and (ii) f_g and f_0 can be calculated, the other quantities being given.

PROBLEMS.

111. Find the maximum charge that can be held on a sphere 200 centimeters in radius, the sphere being surrounded by air of which the electric strength is, say, 33,000 volts per centimeter.

Note.—The maximum electric field intensity at the surface of the sphere being known the electric flux from the sphere can be calculated, etc.

112. A sphere 25 centimeters in diameter is submerged in oil whose inductivity is 2.5 and whose dielectric strength is 60,000 volts per centimeter. Find the maximum charge that can be held on the sphere.

113. Two flat metal plates with 50,000 volts between them are separated by 3 centimeters of oil of which the inductivity is 2, and 4 centimeters of glass of which the inductivity is 6. Find volts per centimeter in each.

114. What maximum voltage can be applied to two plates 10 centimeters apart with air between (strength of air being 33,000 volts per centimeter)? A plate of glass 7 centimeters thick, inductivity 6, dielectric strength 100,000 volts per centimeter is placed between the plates leaving an air layer 3 centimeters thick. What maximum voltage can be applied to the plates without breaking down the air layer?

92. **Electric field between concentric metal spheres.**—A metal sphere of radius R_1 is surrounded by a concentric hollow metal sphere of which the inside radius is R_2 as shown in Fig. 116.

The small sphere carries a charge $+q$ coulombs, and a charge of $-q$ coulombs resides on the inner surface of the outer sphere. The lines of force of the electric field in the region between the spheres are straight lines which meet both spherical surfaces at right angles (these lines of force are shown in part in Fig. 116). It is required to find the intensity f of this electric field in volts per centimeter at any point p distant r centimeters from the common center of the two spheres.

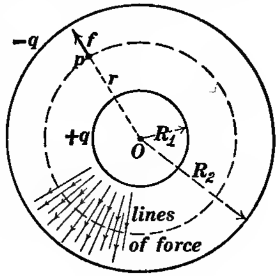

Fig. 116.

The dotted circle in Fig. 116 represents a spherical surface or shell of radius r. The area of this shell is $4\pi r^2$, and the electric flux across it is evidently $4\pi r^2 f$; and, according to Gauss's theorem we must have $4\pi r^2 f = q/B$. Therefore we have

$$f = \frac{1}{4\pi B} \cdot \frac{q}{r^2} \tag{39}$$

where f is the electric field intensity in volts per centimeter at any point distant r centimeters from the center of a sphere on which a charge of q coulombs is uniformly distributed.

93. Potential difference of the two spheres in Fig. 116. Condenser capacity of the two spheres.—The volts per centimeter at the point p in Fig. 116 is $f = \frac{1}{4\pi B} \cdot \frac{q}{r^2}$, and therefore the volts dE along a small radial stretch dr is

$$dE = f \cdot dr = \frac{q}{4\pi B} \cdot \frac{dr}{r^2}$$

and by integrating from $r = R_1$ to $r = R_2$ we get

$$E = \frac{q}{4\pi B}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

whence

$$q = \frac{4\pi B \cdot E}{\frac{1}{R_1} - \frac{1}{R_2}} \tag{40}$$

where E is the electromotive force or potential difference between the two spheres.

Comparing equation (40) with equation (30) of Art. 71 we see that the capacity of a condenser made of concentric spheres with air between is $\dfrac{4\pi B}{\dfrac{1}{R_1} - \dfrac{1}{R_2}}$ farads. If the outer sphere is indefinitely large $1/R_2$ is zero, and the capacity reduces to $4\pi BR_1$ farads. This is usually spoken of as the capacity of an isolated sphere of radius R_1 centimeters.

PROBLEMS.

115. What is the value in coulombs of a concentrated charge which would repel an equal concentrated charge with a force of one dyne at a distance of one centimeter?

Note. See definition of statcoulomb at the end of Art. 69 on page 104. Equation (39) expresses the field intensity f in volts per centimeter at a point distant r centimeters from a concentrated charge of q coulombs. If we place another concentrated charge of q' coulombs at this point, the force F exerted on q by q (a repulsion) will be fq', according to equation (37) on page 125. Therefore

$$F = \frac{1}{4\pi B} \cdot \frac{qq'}{r^2} \tag{i}$$

In terms of what unit is F expressed? This question can always be answered by inserting units of everything in an equation. Now B is farads per centimeter according to equation (31*a*) on page 111. Therefore F must be in $\dfrac{\text{centimeters}}{\text{farads}} \times \dfrac{(\text{coulombs})^2}{(\text{centimeters})^2}$. But from equation (35) on page 113 we get joules = $(\text{coulombs})^2 \div$ farads, and therefore F comes out as joules per centimeter. One joule per centimeter is equal to 10^7 dynes.

116. The earth being at a great distance from all other bodies what is its condenser capacity thought of as an isolated sphere? What amount of charge would there be on the earth if the potential difference between the earth and a very distant region were 1,000,000 volts? What would the electric field intensity be near the surface of the earth in volts per centimeter?

Note.—The *surface* of the earth would here be, undoubtedly, the surface of the atmosphere because the atmosphere is a conductor.

117. A long slim cylinder carries q coulombs of charge per centimeter of length. What is the electric field intensity at a point (in air) distant r centimeters from the axis of the cylinder?

118. A cylindrical conductor one centimeter in diameter is arranged coaxially in a lead pipe of which the inside diameter is 5 centimeters. Adjacent to the rod the insulating material is rubber of which the inductivity k is 2.5; and next to the lead pipe the insulating material is dry paper of which the inductivity is the same, very nearly, as air. The electrical stress in the rubber at the surface of the rod is 10,000 volts per centimeter. What is the electrical stress in the paper at the surface of the lead pipe?

Note.—This problem illustrates the principle of graded cable insulation, namely, the use of insulating material of decreasing inductivity at increasing distances from the core of a cable so as to distribute the total stress (total volts) more evenly through the layer of insulating material. See Franklin and MacNutt's *Advanced Electricity and Magnetism*, pages 156–158.

CHAPTER VII.

THE ATOMIC THEORY OF ELECTRICITY.

94. **Mechanical theory and atomic theory.**—The study of electricity and magnetism as represented in the foregoing chapters is independent of any consideration of the nature of the physical action which leads to the production of electromotive force in a voltaic cell or dynamo; it is independent of any consideration of the nature of the physical action which constitutes an electric current in a wire; it is independent of any consideration of the nature of the disturbance which constitutes a magnetic field; and it is independent of any consideration of the nature of the disturbance or stress which constitutes an electric field. This kind of study of electricity and magnetism may very properly be called *electro-mechanics*.

Simple mechanics is the study of ordinary bodies at rest or in visible motion, and one of the most important ideas in mechanics is the idea of force; but the science of mechanics is not concerned with, and indeed it sheds no light upon the question as to the physical nature of force. Thus, the science of mechanics is not concerned with the question as to the nature of the action which takes place in gas and causes the gas to exert a force on a piston; the science of mechanics is not concerned with the question as to the nature of the action which takes place in the material of a stretched wire causing the wire to exert a pull upon each of the two supports at its ends; the science of mechanics is not concerned with the nature of the action between the earth and a heavy weight which causes the earth to exert a force on the weight; the science of mechanics is not concerned with the nature of the action which takes place between two bodies which slide over each other and produces the force of friction. *It is sufficient for the science of mechanics that these actions are*

what may be called **states of permanency** *of the respective systems.* For example, to say that a gas in a cylinder pushes with a force of 100 "pounds" on the piston thus compressing a spring is to refer to a state of affairs in which there is *a clearly defined and maintained relationship between the condition of the spring and the condition of the gas.*

Similarly it is sufficient for the science of electro-mechanics that the physical actions that underlie electromotive force, electric current, magnetic field and electric field are what may be called *states of permanency.* Thus to say that a dynamo produces a current of 100 amperes in a circuit is to refer to a state of affairs in which there is *a definite and maintained relationship between the dynamo and the circuit,* the dynamo delivers energy at a certain rate and the circuit receives energy at a certain rate, and the circuit exhibits to a definite degree the various effects which are associated with what we call an electric current.

The superficial character of the science of simple mechanics and of the science of electromechanics may be further exemplified as follows: (*a*) An engineer wishes to know the strength of a steel rod, and he finds by test that the rod is broken by a tension of 120,000 "pounds"; but the exact character of the action which takes place in the steel when it is placed under tension is not a matter for consideration, neither does the engineer need to consider (as a part of his test) the action which goes on in the furnace of the boiler that supplies steam to the engine that drives the dynamo that supplies current to the electric motor that drives his testing machine! (*b*) An engineer wishes to know the "strength" of a glass insulator which he is to use to support a wire on a pole-line, and he finds by test that the insulator is broken down or punctured by an electromotive force of 95,000 volts; but the exact character of the action which takes place in the glass when it is subjected to the electromotive force is not a matter for consideration, nor is it necessary to consider the changes which take place in the battery, for example, which may be used to produce the given voltage.

Simple mechanics is concerned with the correlation of what may be called *lump effects*, such as the relationship between the size of a beam and the load it can carry, the size of a fly wheel and the work it can do when stopped, the thickness and diameter of a boiler shell and the pressure it can stand, the size of a submerged body and the buoyant force which acts upon it, the size and shape of the air column in an organ pipe and its number of vibrations per second, the thickness of a glass plate and the electromotive force it can stand, and so forth.

The atomic theory, on the other hand, involves the development of more or less hypothetical conceptions of the minute details of physical action.

The use of the atomic theory in chemistry and in the study of heat phenomena is widely known and understood, but the use of the atomic theory in the study of electricity and magnetism is very recent, and this phase of the atomic theory, which is usually called the *electron theory*, is not very widely known and understood.*

95. Air an electrical conductor.—When a charged gold leaf electroscope is left standing it soon loses its charge and the leaves fall together. This loss of charge has long been known to be due in part to a leakage of the electricity through the surrounding air although usually it is due mostly to a leakage of electricity along the insulating supports. That is to say, air conducts electricity to some extent.

There are a number of influences which cause air (or any gas) to become a fairly good electrical conductor. Thus gas which is drawn from the neighborhood of a flame or from the neighborhood of glowing metal or carbon is a fairly good conductor; gas which has been drawn from a region through which an electric discharge has recently passed is a fairly good conductor; a gas becomes a

* The discussion of the electron theory in this chapter is necessarily very brief. The student who is interested in the subject should read Sir J. J. Thomson's *Conduction of Electricity through Gases*, and Sir Ernest Rutherford's *Radio-active Transformations*.

fairly good conductor under the action of X-rays or under the action of the radiations from radioactive substances.

The conductivity which is imparted to a gas by these various agencies gradually disappears and it may be quickly destroyed by filtering the gas through glass wool or by placing the gas for a few moments between electrically charged metal plates. This effect of filtration seems to show that the conductivity of the gas is due to something which is mixed with the gas, and the effect of the electric field (between the two charged plates) seems to show that this something is charged with electricity and is dragged out of the gas by the electric field. From some such considerations as these the hypothesis was originated that the electrical conductivity of a gas is due to *electrically charged particles* floating around in the gas. These particles are called *ions*, and the process by which a gas is made into a conductor is called *ionization*. This hypothesis has been used extensively and with remarkable success in the study of electrical discharge through gases and in the study of radio-activity.

The *electron* is a negatively charged particle of which the mass is about 1/1835 of the mass of a hydrogen atom. The cathode rays, which are described later, are electrons thrown off from the cathode of the Crookes tube at high velocity, the β-rays from a radio-active substance such as uranium are electrons which are expelled at extremely high velocity from the atoms of the substance.

An *atom* is supposed to be a kind of central nucleus with a number of electrons revolving round it, very much like the solar system which consists of the sun with the planets revolving around it.

A *molecule* is a group of atoms clinging together.

An ordinary atom or molecule is electrically neutral, that is, it has no apparent charge of electricity, the negative charges of the revolving electrons being offset by a positive charge in the nucleus. But when the molecules of a gas are broken up by chemical action, by the effect of high temperature, by X-rays,

or by the corona discharge the *pieces* are electrically charged, and these pieces may be *electrons* or *simple ions*.

A *simple ion* is an atom from which a negatively charged electron has been detached thus leaving the remainder of the atom (the simple ion) positively charged; or a simple ion may be a neutral atom to which an electron has become attached thus giving the atom (the simple ion) a negative charge.

The *compound ion* is an aggregate of two or more atoms or molecules with an extra electron attached to it thus giving it a negative charge or from which a negatively charged electron has been detached thus leaving the aggregate positively charged. In highly rarefied gases electrons and simple ions, only, exist; whereas compound ions may exist in gases at moderate or high pressure.

96. Ionization by collision.—According to the kinetic theory,* a molecule of a gas travels on the average a certain distance between successive collisions with neighboring molecules. This distance is called *the mean free path of a molecule.* The mean free path of an electron is 5 or 6 times as great as the mean free path of a molecule, and the mean free path of an ion is only slightly greater than the mean free path of a molecule.

Consider a body of gas between oppositely charged metal plates as in Fig. 95 or Fig. 96. The electric field between the plates exerts a pull on any electron (or ion) which may be floating around in the gas, and a certain amount of velocity is imparted to the electron (or ion) between successive collisions with the gas molecules. If this velocity exceeds a certain value, the electron (or ion) breaks the molecules of the gas to pieces as it collides with them, and when a molecule of the gas is thus broken to pieces a new electron and a new ion are produced.† This process is called *ionization by collision.*

* The standard general treatise on this subject is O. E. Meyer's *Kinetic Theory of Gases* (translated from the German by R. E. Baynes), Longmans, Green & Co., London, 1899; *Vorlesungen uber Gas Theorie*, Ludwig Boltzman, Leipzig, 1896, is a very important work.

† We are not here concerned with the fact that a single collision may produce more than one new electron and more than one new ion.

Experiment shows that in air at normal atmospheric pressure an electric field of about 13,000 volts per centimeter gives to an electron sufficient velocity between collisions to enable the electron to ionize the air molecules with which it collides, and that an electric field of about 32,000 volts per centimeter gives to a simple ion sufficient velocity between collisions to enable the ion to ionize the air molecules with which it collides.

97. **The electric spark in a gas.**—Let us consider air at normal atmospheric pressure for the sake of definiteness. Figure 117 shows two oppositely charged metal plates A and B, and the electric field between them is approximately uniform.

Suppose that the electric field has been increased to about 13,000 volts per centimeter. Then a stray electron e, as it is drawn towards plate A, will ionize the air along its path p, and each newly formed electron as it travels along with e will ionize the air; *but all of the simple ions thus formed will travel towards B without producing any ionization.* In this case, therefore, ionization takes place only along the path p, and all ionizing action stops when e and the newly formed electrons reach plate A.

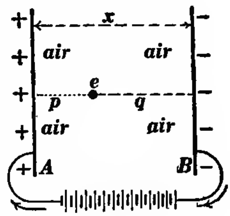

Fig. 117.

Suppose that the electric field has been increased to about 32,000 volts per centimeter. Then any stray electron e, as it is drawn towards plate A, will ionize the air along its path, and, of course, the newly formed electrons will do likewise. In this case, however, the newly formed simple ions also produce new electrons and simple ions as they travel towards plate B. It is evident, therefore, that the ionizing action does not come to an end, but increases indefinitely all along the entire path pq, and the result is the production of a spark along pq. The beginning of the intense ionization along the path pq is here attributed to the stray electron e, but the action may of course be started by a stray simple ion because the field intensity is great enough to enable a simple ion to produce new ions and electrons by collision.

According to the kinetic theory of gases, the mean free paths of the electrons and simple ions are doubled when the pressure (or density) of the gas is halved, and with doubled mean free path sufficient velocity to ionize by collision is produced by an electric field one half as intense. Therefore the dielectric strength of air should be proportional to its pressure (temperature being constant). This, in fact, is the case. Thus the dielectric strength of air at normal atmospheric pressure is about 32,000 volts per centimeter, at a pressure of 10 atmospheres it is about 320,000 volts per centimeter, and at a pressure of 0.1 atmosphere it is about 3,200 volts per centimeter. The dielectric strength of air reaches a minumim, however, at a pressure of about 2 millimeters of mercury, and increases when the pressure falls below this value.

The idea of specific dielectric strength is based on the assumption that the electromotive force required to produce a spark is proportional to the length of the spark, so that the quotient, volts divided by spark length, may be a constant for a given material. This is approximately true in gases under moderate or high pressure,* but when the pressure is very low *a greater electromotive force is required to strike across a short gap than is required to strike across a long gap.*†

98. The corona discharge.—The curved lines in Fig. 94 show, approximately, the mode of distribution of the electric field between two parallel wires; the plane of the paper is at right angles to the wires, and the small circles represent the wires in section. The electric field is much more intense near the wires where the lines of force are crowded together than it is at greater distances from the wires where the lines of force spread more widely apart. If the wires are very fine the electric field may be very intense in a very small region around each wire.

Imagine the electromotive force between a given pair of wires to be increased until the field intensity near the wires is about

* And, of course, between flat electrodes. See Art. 76.

† This behavior of a gas at low pressure is fully explained by the atomic theory. See J J. Thomson's *Conduction of Electricity through Gases*, pages 430–527.

13,000 volts per centimeter. Then a stray electron starts an ionizing action which quickly comes to an end either when the original electron and the newly formed electrons reach the surface of the wire (the positively charged wire) or when the original electron and the newly formed electrons pass out from the neighborhood of the negatively charged wire into a region where the field intensity is decidedly less than 13,000 volts per centimeter.

Imagine the electromotive force between the wires to be increased until the field intensity near each wire is about 32,000 volts per centimeter. Then a stray electron or a stray simple ion near either wire will start an ionizing action which will *continue* indefinitely. But this ionizing action does not *increase* indefinitely as in the formation of a spark as described in connection with Fig. 117. The reason for this limitation is as follows: Let us consider the c o r o n a around the positively charged wire, for example. In the region *rr* near the wire in Fig. 118 the electric field is sufficiently intense to enable electrons and ions both to produce new electrons and ions by collision. The electrons (negatively charged) travel to the surface of the wire where they are absorbed, but the positively charged ions, having to travel far through the surrounding region *RR*, accumulate in this region in large numbers as an extensive cloud of positive charge surrounding the region *rr* on all sides as indicated by the dots in Fig. 118. This accumulation of a cloud of positive charge (positive ions), as shown by the dots in Fig. 118, reduces the intensity of the electric field near the wire because the lines of force do not all emanate from the positively charged wire but many of these lines originate in the cloud of positive charge. Excessive production of ions in the region *rr* in Fig. 118 would

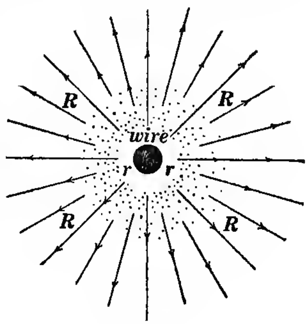

Fig. 118.

thus reduce the field intensity in this region so that no more ions would be formed and therefore excessive ionization in the region *rr* cannot take place.

There is a sharply defined value* of the voltage between parallel wires for which the electric field intensity near the wires is sufficiently intense to maintain steady ionization (by electron collisions and ion collisions). The corona appears when the voltage between the wires reaches or slightly exceeds this value, and for greater values of voltage the corona spreads out farther and farther from the wires.

99. The Cottrell process for the precipitation of dust and smoke particles.—A fine wire is stretched along the axis of a metal tube; a steady (*not* alternating) voltage high enough to produce corona discharge around the fine wire is connected between wire and tube; and the result is that the particles of smoke or dust are deposited on the walls of the tube and to a very slight extent on the wire.

The most satisfactory arrangement for demonstrating the Cottrell process is shown in Fig. 119. A fine wire AA, sup-

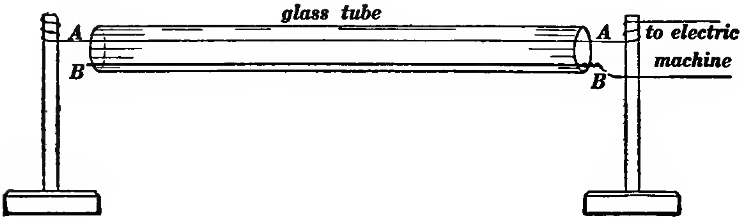

Fig. 119.

ported by two insulating glass posts, is stretched through a large glass tube, and a piece of strap iron BB is laid along the bottom of the tube as shown in the figure. The fine wire and iron strap are connected to the terminals of a Toepler-Holtz

* Depending on size of wires and their distance apart. The mathematical theory of the distribution of electric field between parallel wires is developed in Chapter VIII of Franklin and MacNutt's *Advanced Electricity and Magnetism*, published by Franklin and Charles, Bethlehem, Pa.

influence machine, and the smoke to be cleared is blown in at one end of the tube. The effect is most strikingly shown by short-circuiting the influence machine by means of a small metal rod, filling the tube with smoke, and then suddenly removing the short circuit.

The Cottrell separator works almost equally well whether the wire be made positive or negative. If the wire is positive, great numbers of simple ions with their positive charges travel out from the corona, these ions attach themselves to the particles of dust or smoke, these particles thus become positively charged, and the electric field between wire and tube drags the positively charged particles to the negatively charged wall of the tube. If the wire is negative, great numbers of electrons with their negative charges travel out from the corona and attach themselves to the smoke or dust particles, and the particles thus negatively charged are attracted by and travel to the positively charged wall of the tube.

100. **The ozonizer.**—The ozonizer is a device for converting oxygen into ozone. The essential features of the device are shown in Fig. 120. Two metal plates *A* and *B*, with a glass plate between, are connected to the terminals, *ab*, of the high-voltage coil of a step-up transformer, a blast of air is blown through between the plates, and a portion of the oxygen of the air is converted into ozone.

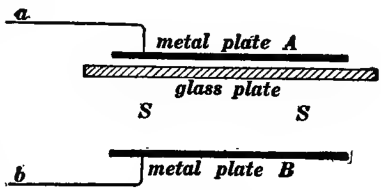

Fig. 120.

Essential features of ozonizer (*a* and *b* connected to high voltage supply of alternating current).

The high electromotive force (alternating, of course) between the metal plates would produce a spark discharge from metal plate to metal plate in the absence of the glass plate, but the glass plate, if it is not punctured, prevents the formation of one single path of indefinitely intense ionizing action (like the path

pq in Fig. 117 as explained in Art. 97), so that moderately intense ionizing action takes place throughout the entire region *SS*.

The conversion of oxygen into ozone takes place as follows: Ordinary oxygen has two atoms in the molecule. These molecules are split into atoms in the ionizing process, some of these atoms recombine as triplets, and these triplets are ozone molecules.

101. Emission of electrons by hot bodies.*—Electrons are emitted in great numbers by a hot body if the temperature is sufficiently high. Thus, Fig. 121 represents an ordinary tungsten lamp filament *c* which is heated to a high temperature by the battery *A*. The hot filament emits electrons at a rate sufficient to carry a few hundredths of a coulomb of negative charge per second from *c* to *a*, that is to say, at a rate sufficient to permit the battery *B* to produce a few hundredths of an ampere of current in the direction of the arrow; the travel of negative charge from *c* to *a* is equivalent to the travel of positive charge from *a* to *c*, and the direction of a current is thought of as the direction of travel of positive charge.

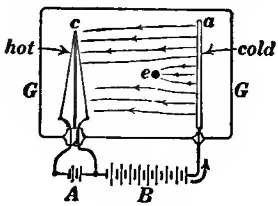

Fig. 121.

If the electromotive force of the battery *B* is large and if the bulb *GG* contains gas, the gas molecules are ionized by collision, and the additional electrons and ions thus produced carry a greatly increased current from *a* to *c*. Indeed, with gas in the bulb an indefinite increase of ionization and indefinite increase of current take place if the voltage of battery *B* is high. *The simplest practical applications of electron emission by a hot body depend upon the absence of gas in the bulb GG and the consequent elimination of ionization of the gas molecules by collision.*

The cold electrode in Fig. 121 is an anode (positively charged)

* This subject has been extensively investigated particularly by Professor O. W. Richardson, who has brought most of his researches together in his recent book *The Emission of Electricity from Hot Bodies*, Longmans & Co., London, 1915.

and the lamp filament is a cathode. The fine lines represent the lines of force of the electric field between *a* and *c*. Some of these lines of force terminate on the electrons which are en route from *c* to *a*. Thus three of the lines of force in Fig. 121 are shown ending on an exaggerated electron *e*. Therefore if there are enough electrons en route at each instant *all* the lines of force from *a* will end on the moving electrons, and no lines of force will reach the cathode *c*; that is to say, the electric field in the neighborhood of *c* will be zero. Under these conditions the electron emission of the filament *c* might be greatly increased (by raising its temperature) without any increase of current; because the electrons are emitted by *c* at very low veloicties and, unless they are drawn away from *c* by an electric field they form a comparatively stagnant cloud around *c*.

The emission of electrons by a hot metal surface is made use of in several important practical appliances namely, (*a*) The Coolidge X-ray tube, (*b*) The vacuum tube rectifier, (*c*) The pleiotron, and (*d*) The tungar.

102. The Coolidge tube.—The essential features of the *Coolidge tube* are shown in Fig. 122. A high-voltage supply maintains a

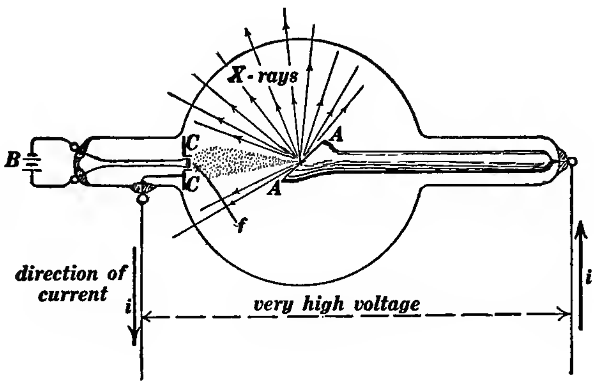

Fig. 122.

very intense electric field between a metal cathode *CC* and a massive tungsten anode *AA*. A fine filament of tungsten wire,

f, like an incandescent lamp filament, is heated by the insulated battery *B*, and it is connected to and forms part of the cathode *CC*. The electrons which are given off by the hot filament *f* are pulled across* from *C* to *A* by the electric field in a slightly converging stream as indicated by the fine dots,† and they impinge on a small spot on the face of the tungsten anode. The stream of electrons constitutes what is called the *cathode rays*, and the fine radiating lines represent the X-rays which radiate from the spot where the cathode stream strikes the tungsten plate *AA*.

With the Coolidge tube the current through the tube can be controlled by changing the temperature of *f* thus changing the number of electrons per second which are available for carrying negative charge from *C* to *A*, and the supply voltage *E* can be high or low as you please. The higher the voltage *E* the more penetrating the X-rays, and the greater the current *with a given voltage* the more intense (the greater the energy value of) the X-rays. With the Coolidge tube these two qualities of the X-rays can be adjusted independently of each other.‡

Current is carried through the Coolidge tube by the negative electrons, and negative electrons are supplied only at the plate *CC* (at the filament *f*).§ *Therefore current can flow through the Coolidge tube in one direction only*, in the direction corresponding to the travel of negative charge from *C* to *A*, as indicated by the arrows *ii*. The Coolidge tube can, indeed, be operated by connecting *C* and *A* to the high-voltage coil of a step-up

* See Art. 80. The charged ball in Fig. 96 is pulled towards plate *A* or towards plate *B*, depending upon whether the charge on the ball is positive or negative.

† The converegnce of this stream of electrons is due to the breadth of the cathode *CC*.

‡ With a given voltage the current is limited, and, under ordinary conditions, partly determined by the distribution of negative charge on the electrons which are *en route* between *C* and *A*, and because of this space-charge effect the statement that *intensity* and *penetration* of the X-rays can be independently controlled is not strictly true, although practically it is true.

§ The plate *AA* may become very hot on account of the bombardment by the particles in the cathode stream, and if *AA* does thus become hot it gives off electrons.

transformer. When the alternating voltage is in one direction a stream of electrons is carried across from C to A and X-rays are produced, but when the alternating voltage is reversed no current can flow.

103. The vacuum-tube current-valve or rectifier.—When the Coolidge tube is operated by connecting cathode and anode, C and A in Fig. 122, to high-voltage alternating supply mains, current can flow in one direction only, as above explained. That is to say, the Coolidge tube acts like a valve. When this valve action alone is desired the cathode is made of a plane zig-zag or grid of fine tungsten wires which are kept at a high temperature by a small auxiliary battery connected as in Fig. 122, and the cold anode is made of two flat metal plates, one on each side of the cathode. *Current can flow only in the direction corresponding to the movement of negatively charged electrons from hot cathode to cold anode.*

104. The use of the vacuum-tube current-valve as a receiver or detector in wireless telegraphy.—The essential features of a wireless receiving station using a vacuum-tube current-valve are shown in Fig. 123. The electric waves from the sending

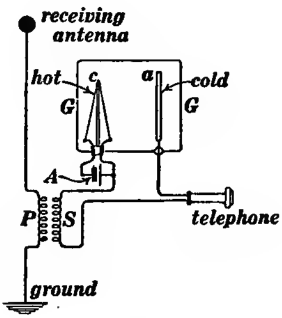

Fig. 123.

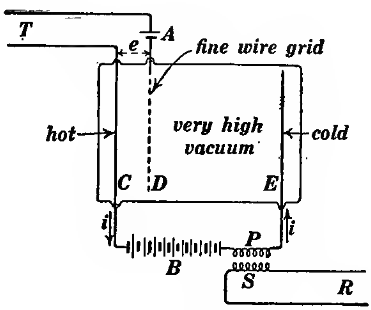

Fig. 124.

antenna (see Fig. 90a) produce high-frequency alternating current in the receiving antenna, high-frequency current is induced in the secondary coil S, every alternate half-wave of this high-

frequency current flows through the current valve, and these half-waves of the alternating current blend into a pulse of direct current in the telephone.

105. **The pleiotron** consists of a cold electrode or plate E, Fig. 124, a fine wire grid D and a hot electrode C, all enclosed in a very highly exhausted glass bulb. The hot electrode C is a tungsten filament and it is heated by an auxiliary battery exactly as in case of the Coolidge tube but these details are not shown in Fig. 124 which shows the pleiotron arranged as a telephone amplifier or repeater. The two wires T deliver a very weak voltage-current wave from a distant telephone transmitter, and the two wires R receive from the pleiotron a greatly intensified voltage-current wave which actuates a distant telephone receiver.

To understand the action of the pleiotron as a telephone amplifier or repeater let us consider the action of the fine wire grid as follows: Electrons are thrown off from the hot electrode or filament, and a large fraction of these electrons shoot through

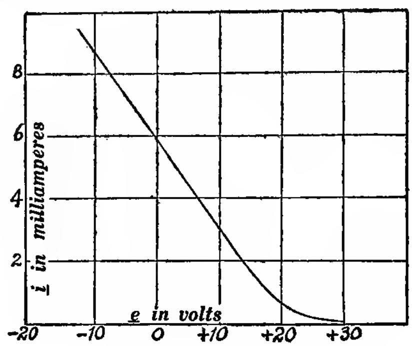

Fig. 125.

the meshes of the grid and thus carry negative charge from C to E which means a flow of battery current in the direction of the arrows ii. When the voltage e between C and D is in the direction to charge the grid negatively, the negative charge on the grid repels the electrons, and as the value of e increases the negative charge on the grid increases, the number of electrons

per second which shoot through the grid decreases, and the current i decreases. This relation between e and i is shown by the curve in Fig. 125. The battery current i in Fig. 124 therefore rises and falls with the voltage e, and the this varying battery current, in flowing through the primary coil of the transformer PS, induces in the secondary coil S an electromotive force which increases and decreases with e thus reproducing the original wave.

106. The use of the pleiotron for exciting and maintaining electric oscillations.—Figure 126 shows a high-voltage battery B and a pleiotron arranged to maintain the electrical oscillations

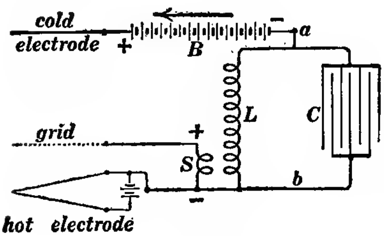

Fig. 126.

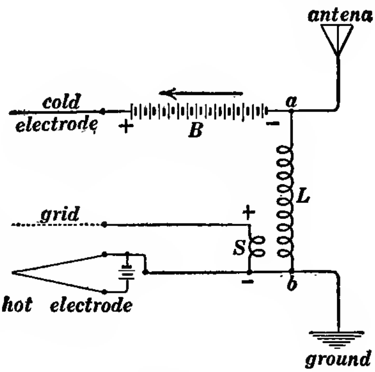

Fig. 127.

of the circuit consisting of a condenser C and an inductance L, and Fig. 127 shows a high-voltage battery B and a pleiotron arranged to maintain, the electrical oscillations of the sending antenna of a wireless station.

Let us assume that the antenna in Fig. 127 is already oscillating, then terminal a is at very low potential for a short time during each oscillation, and all that is required to maintain the oscillations is to draw charge out of a every time a falls to a very low potential.

The tickler coil S is a secondary coil under the influence of L as a primary and the terminals of S are connected to hot fila-

ment and grid so as to charge the grid *positively* while a is at *low potential* and *vice versa*, thus permitting battery current to flow when a is at low potential so as to draw charge out of a as stated.

The oscillations of the antenna very quickly rise to a steady maximum, and when this condition is reached energy is dissipated in and radiated by the antenna as fast as energy is delivered by the battery and pleiotron. Therefore increased damping (increased loss of energy for a given violence of oscillation) causes the violence of the oscillations to decrease, and *vice versa*.

107. **The wireless telephone.**—At the sending station the wireless antenna is kept oscillating at a very high frequency by means of a pleiotron arranged as in Fig. 127, and a device called a *modulator* is used to control the violence or intensity of these antenna oscillations so that their intensity will increase and decrease with the to and fro motion of the diaphragm of an ordinary telephone transmitter. One arrangement of this modulator is described in connection with Fig. 128.

The result of the modulation of the oscillations of the antenna at the sending station is to cause the antenna to give out electric waves whose intensity rises and falls with the to and fro motion of the diaphragm of the transmitter. These waves act upon a receiving antenna arranged as in Fig. 123, and the rectified current in the telephone in Fig. 123 rises and falls with the to and fro motion of the distant transmitter diaphragm thus reproducing the original sound.

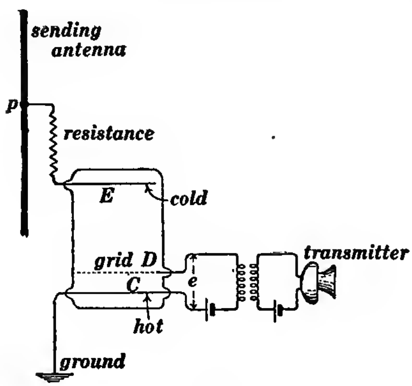

Fig. 128.

The modulator.—The simplest arrangement for modulating the oscillations (very high frequency oscillations) of the sending

antenna in wireless telephony is shown in Fig. 128. A pleiotron is connected in series with a moderate resistance and bridged across the sending antenna from the point p to ground as shown in Fig. 128. As the voltage e rises and falls with to and fro motion of the transmitter diaphragm more or less current can flow from p to ground, and this flow of current represents more or less loss of energy or more or less damping of the antenna oscillations. Therefore the violence of the antenna oscillations rises and falls with the to and fro motion of the diaphragm of the transmitter.

Remark.—Everyone who has worked at wireless telegraphy knows that an almost endless variety of connection schemes can be used. Only the simplest schemes are considered in this chapter.

108. The tungar.—The completely exhausted current valve which is described in Art. 103 is limited in its current carrying capacity because the only electric carriers are the electrons which are emitted by the hot filament. Indeed the largest highly exhausted current values that have been hitherto made commercially carry a maximum current of a few thousandths of an ampere.*

The tungar is a current valve in which there is residual gas, and in which the residual gas is ionized by collision thus providing for a very great increase in the number of carriers, and giving a correspondingly increased current carrying capacity.

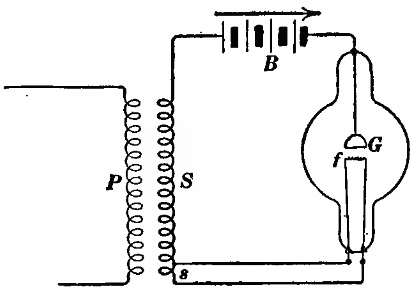

Fig. 129.

The transformer PS is designed so as to have a considerable choking effect on the alternating current and thus protect against excess flow of current.

The tungsten filament f, Fig. 129, is heated by current delivered by a small section s of the transformer secondary. This

* Experimental tubes have been made to carry several amperes.

hot filament gives off electrons continuously, and when the alternating supply voltage is in the direction of the arrow then electrons are drawn towards the graphite electrode G, the gas is ionized by collision, and a current of several amperes is carried partly by the original electrons but chiefly by the newly formed electrons and positive ions.

When the supply voltage reverses its first effect (while it is still small in value) is to sweep all of the residual electrons and ions out of the space between f and G, and as it grows larger it drives all newly emitted electrons back into the filament f so that no electrons or ions are available for starting ionization by collision and consequently no current can flow.

The gas in the tungar bulb is argon.

109. The Crookes tube. Cathode rays and canal rays.*— The older form of Crookes tube is a glass bulb with two electrodes sealed in, and the bulb is exhausted until the pressure of the residual gas is about a millionth of an atmosphere, so that the mean free path of the electrons and positive ions may be several centimeters. When a fairly high electromotive force (ten thousand volts or more) is connected to the electrodes, continuous ionization of the residual gas takes place because of collisions, the electrons travel in nearly straight paths away from the cathode (the negatively charged electrode) constituting what are called *cathode rays*, and the positive ions travel in nearly straight paths towards the cathode constituting what are called *canal rays*. The cathode rays produce luminescence of the walls of the bulb where they strike, and the straight-line travel of the cathode rays is shown by the fact that any object placed in front of the cathode casts a shadow (a spot where there is no luminescence) on the wall of the bulb. The canal rays show themselves as luminous streaks behind the cathode when the cathode is perforated.

Cathode rays are deflected by a magnetic field or by an electric

* See J. J. Thomson's *Conduction of Electricity through Gases* for a full discussion of this topic.

field in the same way that the β-rays from radium are deflected, and canal rays are deflected in the same way that α-rays from radium are deflected. This matter is discussed very briefly in Art. 111.

110. Radio-activity.*—The chemical elements uranium, thorium, and radium and their compounds have the property of making a surrounding gas an electrical conductor. Thus, one ten-millionth of a gram of radium bromide which is left as a residue upon a metal plate by evaporating a small quantity of a dilute solution of radium bromide on the plate, causes a gold leaf electroscope to be discharged in a few seconds when the radium-covered plate is held near to the metal plate of the electroscope. Uranium and thorium have the same effect but the discharge which they produce is not so rapid unless a large quantity of material is employed. This property of these metals and of their compounds is called *radio-activity*, a name which originated because of the peculiar radiations which are given off by these substances and to which the discharging action is due. These radiations are of three distinct kinds, which are called the α-rays, the β-rays, and the γ-rays, respectively. The γ-rays penetrate through a foot or more of solid metal or through many feet of air; the β-rays penetrate through a moderate thickness of a light metal, such as aluminum; whereas the α-rays are stopped by a very thin layer of aluminum or by a layer of air two or three inches in thickness.

The α-rays consist of positive ions each about four times as massive as a hydrogen atom, and they are projected from the radioactive substance at a velocity of about 20,000 miles per second. After traveling two or three inches through the air, the velocity of these α-particles is reduced to so low a value as to render them no longer perceptible by their ionizing effects.

* The student is referred to the following books for a full discussion of radioactivity: *Radioactivity*, by E. Rutherford, Cambridge, 1905 (second edition); *Radioactivity*, by Frederick Soddy, London, 1904; and *Radioactive Transformations*, by E. Rutherford, New York, 1906. See also Rutherford's *Radioactive Substances and their Radiations*, Cambridge, 1913.

The β-rays consist of electrons (negative ions) each about 1/800 as massive as a hydrogen atom, and they are projected from the radio-active substance at a velocity which in some cases is nearly as great as the velocity of light (186,000 miles per second). The β-particles also have the property of ionizing the gas through which they pass but not to so great an extent as the α-particles, and they travel several feet through the air before their velocity is reduced to so low a value as to render them no longer perceptible by their ionizing effects.

The γ-rays are essentially like X-rays but of shorter wave-length and greater penetrating power. The γ-rays also have the power of ionizing a gas.

The present hypothesis regarding radio-activity is that the atom of a substance is a system of excessively small negatively charged particles, called electrons, revolving around a nucleus containing positive charge, the atom of each element being a characteristic group or system. These systems of electrons (atoms) are supposed to be to some extent unstable, and when instability occurs, the system (atom) collapses into a new configuration, and at the same time expels one or more positively or negatively charged particles which constitute the α-rays and the β-rays.

111. **Determination of velocity and mass of α-particles and of β-particles.**—The following experimental methods for deter-

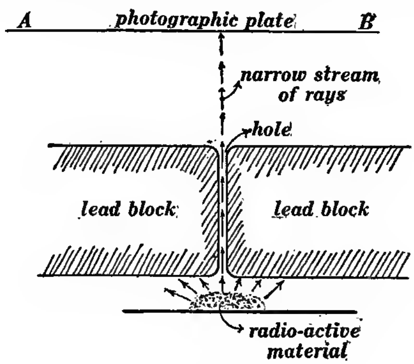

Fig. 130.

mining the velocity and mass of α and β particles have been applied to the particles which constitute the canal rays and the cathode rays in a Crookes tube, but the methods are here described as applying only to α- and β-particles from radium or other radioactive substance.

A narrow stream of rays from a radio-active substance may be obtained by the arrangement which is shown in Fig. 130, and the point where the stream strikes the photographic plate is shown when the plate is developed.

Figure 131 shows the effect of an electrical field on a thin stream of rays from a radio-active substance. The horizontal

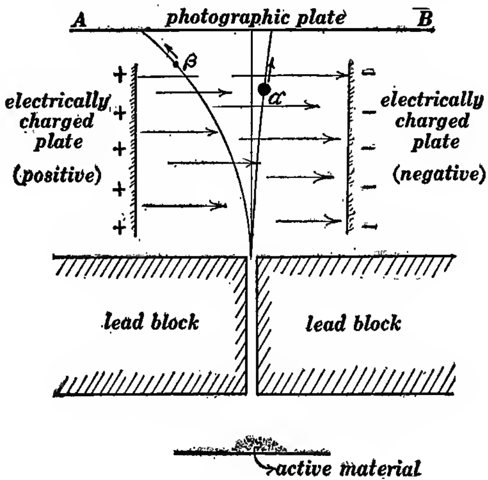

Fig 131.

arrows represent the lines of force of the electric field. The electrical field deflects the α-particles in the direction of the field, it deflects the β-particles in the opposite direction, and it does not deflect the γ-rays at all. The amount of deflection of α-particles and of β-particles may be determined by developing the photographic plate and making the necessary measurements.

The deflection of α-particles and β-particles by the magnetic field is shown in Fig. 132 in which the horizontal arrows represent the lines of force of the magnetic field.

The determination of the velocity of the α-particles and β-particles is somewhat analogous to the following method for determining the velocity of a cannon ball. The curved line

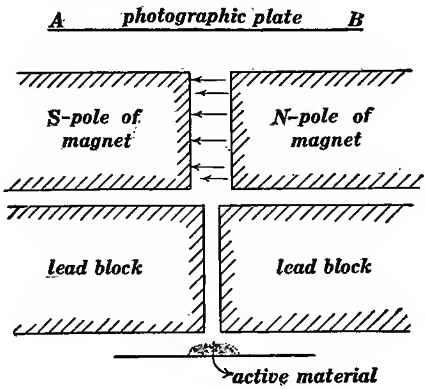

Fig. 132.

The α-particles are deflected towards the reader, and the β-particles are deflected away from the reader.

in Fig. 133 represents the path of the ball, D being the horizontal distance traveled by the ball and d being the vertical distance fallen by the ball under the deflecting force of gravity. If D

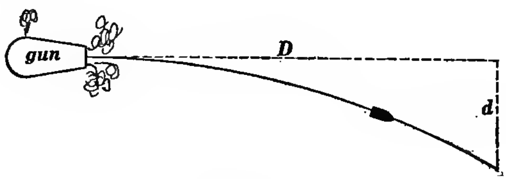

Fig. 133.

is known and d observed, the velocity of the ball is given by the equation

$$v^2 = \frac{gD^2}{2d} \tag{i}$$

in which g is the acceleration of gravity.

Action of the electric field on a moving charged particle.— Consider a charged particle moving upwards through an electrical

field as indicated in Fig. 134. Let q be the charge on the particle in abcoulombs, and let f be the intensity of the field in abvolts per centimeter. Then the force F in dynes pulling

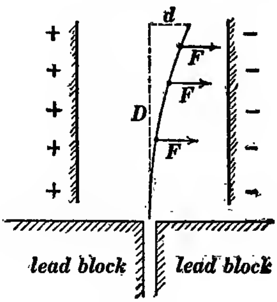

Fig. 134.

Electric field from left to right.

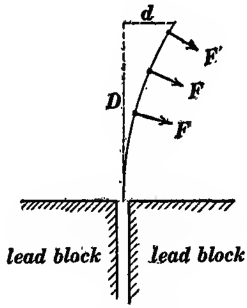

Fig. 135.

Magnetic field towards reader.

sidewise on the particle is fq so that the sidewise acceleration of the particle is fq/m, where m is the mass of the particle in grams. It is evident that the particle moves in the same kind of path as the cannon ball in Fig. 133 (resistance of air zero). Therefore using fq/m for g in equation (i), we have

$$v^2 = \frac{D^2fq}{2dm}$$

or

$$\frac{q}{m} = \frac{2dv^2}{D^2f} \tag{ii}$$

When the velocity v has been determined, as explained below, this equation gives the value of the ratio q/m.

Action of the magnetic field on a moving charged particle.—Figure 135 represents a charged particle moving upwards through a magnetic field. The moving particle is equivalent to an electric current and the side-wise force F is equal to qvH where q is the charge on the particle, v is the velocity of the particle and H is the intensity of the magnetic field in gausses. The force F is always at right angles to v and therefore the path of the par-

ticle is a circle, and the radial acceleration is F/m or qvH/m where m is the mass of the particle. But the acceleration of a particle moving at velocity v in a circular path of radius r is v^2/r, and therefore

$$\frac{qvH}{m} = \frac{v^2}{r}$$

or

$$\frac{q}{m} = \frac{v}{rH} \tag{iii}$$

When measurements have been taken from which r can be calculated, when H is known, and when v has been determined as explained below, this equation also gives the value of the ratio q/m.

Calculation of velocity of moving particle.—Equating the two values of q/m from equations (ii) and (iii) and solving for v, we get

$$v = \frac{D^2 f}{2drH} \tag{iv}$$

The significance of D and d may be understood by comparing Figs. 133 and 134, and both may be easily measured; also f, r and H are easily determined, and therefore, when the necessary measurements have been made, equation (iv) gives the value of v. In this way it is found that the velocity of the α-particles from radium is about 20,000 miles per second, and the velocity of the β-particles from radium is about 160,000 miles per second.

The mass of a moving particle increases with its velocity, and the above equations should be modified to take this increase of mass into account; the purpose of the above discussion, however, is to set forth only the simplest ideas of the subject. A very good discussion of the methods for determining the values of m and q is given in a very recent book, *The Electron*, Professor R. A. Millikan.

112. Electron theory of conduction in metals, contact potential-differences, and thermo-electromotive forces.—The electron theory has given a wonderfully consistent and accurate inter-

pretation o. many of the phenomena of the discharge of electricity through gases, whereas the application of the electron theory to metallic conduction, to contact potential-differences and to thermo-electromotive forces is by no means entirely consistent or accurately interpretative, and the following discussion is justified chiefly by the fact that the phenomena themselves are familiar. The older atomic theory, likewise, has given a wonderfully complete and accurate interpretation of the properties of gases, whereas the application of the atomic theory to liquids and solids is by no means complete or accurately interpretative, and yet everyone nowadays accepts the atomic theory as applying to liquids and solids.

The fundamental hypothesis on which the following discussion is based is that a block of metal contains a great number of free electrons (negatively charged particles) which wander about in the block of metal very much as the atoms of a gas wander about in a containing vessel.

The electric current is carried through a wire by a steady drift of the free electrons, and heat is generated by the impact of the electrons with the atoms or molecules of the metal, an electron being set in motion again after each impact by the electromotive force along the wire.

When the metal is heated the irregular to and fro motion of the free electrons is increased, an increased *pressure* (electron pressure) is produced by this increased motion very much as the pressure of a gas is increased by the increased molecular motion when the gas is heated, and when the temperature is very high some of the electrons escape from the metal as described in Art. 101.

Contact difference of potential.—Figure 136 represents two blocks of different kinds of metal AA and BB welded together along cc. Metal A contains more free electrons per cubic centimeter than metal B, therefore the electron pressure in A is greater than the electron pressure in B, therefore electrons tend to drift from the high pressure region P to the low pressure region p, the effect of this drift is to leave an excess of positive

charge in A and produce an excess of negative charge in B, and the drift continues until these charges become large enough to balance the difference of electronic pressure. Thus Fig. 136 shows an electron e being pulled back from B towards A by the electric forces.

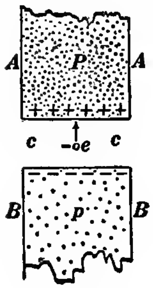

Fig. 136.

c c represents an actual contact or junction of A and B very greatly magnified and idealized.

When the balanced condition is reached there is a definite electromotive force or potential difference between AA and BB which is called a *contact potential difference.* Such contact potential differences have long been known to exist, they are quite small, only a few tenths of a volt, and they cannot be detected by an ordinary voltmeter because they always balance out when different metals are connected in a circuit (temperature being everywhere the same).

The thermo-element and thermo-electromotive force.*— When a circuit is made of two metals as indicated in Fig. 137, a

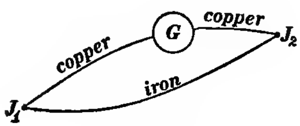

Fig. 137.

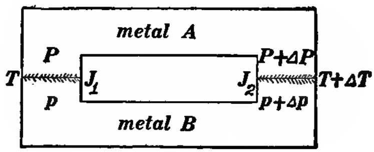

Fig. 138.

current flows around the circuit when the two junctions J_1 and J_2 are at different temperatures. In fact a definite electromotive force acts around the circuit when the temperatures of J_1 and J_2 are given. This electromotive force, which can be measured by a galvanometer G is called a *thermo-electromotive force,* and the circuit of two metals is called a *thermo-element.*

Figure 138 represents a circuit formed of two metals A and

* A fairly complete discussion of thermo-electromotive forces is given in Nichols and Franklin's *Elements of Physics,* Vol. 2, pages 216–221. See also J. J. Thomson's *Elements of Electricity and Magnetism,* pages 506–518.

B, one junction J_1 being at temperature T and the other junction J_2 being at temperature $T + \Delta T$. The electronic pressure in metal A increases from P to $P + \Delta P$, and the electronic pressure in metal B increases from p to $p + \Delta p$ due to increase of temperature from T to $T + \Delta T$ as indicated in the figure, and as in the case of gases $\frac{\Delta P}{\Delta p} = \frac{P}{p}$. That is to say the increase of electronic pressure ΔP is larger than the increase of electronic pressure Δp, consequently the contact potential difference at J_2 is *not* equal to the contact potential difference at J_1 and, as a result,* current flows round the circuit.

* This statement ignores the potential drop along A due to changing electronic pressure along A, and it ignores the potential drop along B due to changing electronic pressures along B. No attempt is here made to give a complete discussion. Indeed a complete discussion does not lead to results which are in accordance with all the known facts.

APPENDIX A.

THE MAGNETISM OF IRON.

1. **Magnetizing force in iron.**—When an iron rod is placed in and parallel to a magnetic field the rod is magnetized, and after the rod is magnetized its free poles modify the previously existing magnetic field. *The actual magnetic field which acts upon and magnetizes the rod may be very different from the original field.* For example an iron rod 20 centimeters long is placed inside of a long coil where the magnetic field, before the rod is put in place, has an intensity of 40 gausses. Let us suppose the rod to be magnetized so that each of its poles has a strength of 1,600 units. The actual magnetic field is now due to the combined action of the coil and the free poles of the rod, and near the middle of the rod the intensity of this actual field is 40 gausses *minus* 32 gausses, or 8 gausses, because, according to Art. 11 of Chapter I, the poles of the rod produce a field intensity of 32 gausses near the middle of the rod and in a direction opposite to the field due to the coil.

The "magnetizing force" or magnetizing field means always the net intensity of field due not only to outside agencies but due also to the free magnet poles on the magnetized iron itself.

There are two cases in which free magnet poles are without appreciable influence, namely, (*a*) When a very long and slender iron rod is magnetized, and (*b*) When the magnetic circuit is wholly of iron so that the magnetic flux never passes from iron to air (a north pole) or from air to iron (a south pole). In case *a* the poles are so weak and so far away from the middle portions of the rod that their influence is negligible, and in case *b* there are no free poles.

2. **Flux density in iron. Definition of magnetic permeability.** —Consider a long coil of wire (a long tube wound with wire

over its entire length). The intensity of the magnetic field inside of this coil in gausses is

$$H = 4\pi \frac{Z}{l} \cdot I \tag{1}$$

where Z is the number of turns of wire on the long tube, l is the length of the tube in centimeters, and I is the current in abamperes.

Let q be the sectional area of the opening or bore of the tube. Then when the tube is filled with air, or any ordinary non-magnetic material, the magnetic flux through the tube is qH, according to Art. 12 of Chapter I, and the flux per unit area (the *flux density*) is, of course, H.

If the long tube is filled with iron the magnetic flux through the tube is many times as great and the flux per unit area (the *flux density*) is correspondingly great. Indeed we may write

$$B = \mu H \tag{2}$$

where H is the "magnetizing force" or magnetizing field, B is the flux density in the iron rod, and μ is what is called the *permeability* of the iron. The permeability of a given sample of iron is not constant but it grows less and less as the flux density B increases (as the iron approaches what is called magnetic saturation). The permeability of air and of all ordinary non-magnetic materials is unity.

The accompanying table gives the corresponding values of B and H for wrought iron, for cast iron and for soft cast steel.

TABLE.

Magnetic Properties of Iron and Steel.

Wrought Iron (Hopkinson).			Cast Iron (M. E. Thompson).			Soft Cast Steel.		
H	B	μ	H	B	μ	H	B	μ
10	12,400	1,240	10	5,000	500	10	9,700	970
20	14,330	716	20	6,600	330	20	13,380	669
30	15,100	503	30	7,290	246	30	13,500	483
40	15,550	389	40	7,850	195	40	15,250	381
50	15,950	319	50	8,360	169	50	15,840	317
60	16,280	271	60	8,800	146	60	16,300	272
70	16,500	235	70	9,200	131	70	16,750	239

3. The magnetic circuit. Definition of magnetomotive force.—The path of any portion of magnetic flux is always a closed path or circuit, and in many cases in practice this closed path or circuit is nearly all of iron. Therefore as the simplest case let us consider a magnetic circuit, so called, which is wholly of iron. Thus the iron rod which is shown in Fig. 1 is a magnetic circuit, *the magnetizing action of the coil on the rod depends upon the average value of the magnetizing field, the average value of* H, *along the rod*; **and the product** lH **is called the magnetomotive force around the circuit which is formed by the rod,** *where* l *is the length of the rod (distance around the circuit) and* H *is the average magnetizing field (component of actual field parallel to the rod at each point).*

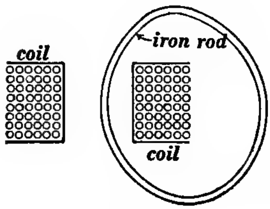

Fig. 1.

This definition of magnetomotive force applies not only to an entire circuit, but to any portion of a circuit. Thus lH is the magnetomotive force acting on any portion of a magnetic circuit where l is the length of the portion and H is the average magnetizing field along and parallel to the portion.

4. Magnetomotive force of a coil.—The magnetomotive force around any circuit or path which links with a coil or winding of wire is

$$F = 4\pi ZI \tag{3}$$

where Z is the number of turns of wire in the coil and I is the current in the coil in abamperes. This equation is derived in Art. 7 of this Appendix.

5. Calculation of the magnetomotive force required to produce a specified amount of magnetic flux around a given magnetic circuit.

(*a*) Divide the specified amount of flux by the sectional area of each portion of the given magnetic circuit, wrought iron,

cast iron, steel or air as the case may be. This gives the flux density B in each portion of the circuit.*

(*b*) Knowing B for each part of the magnetic circuit, take from the above table the corresponding value of H for each part of the circuit. In an air gap the value of H is equal to B.

(*c*) Multiply the value of H for each portion of the magnetic circuit by the length l of that portion in centimeters. This gives the magnetomotive force required for each portion.

(*d*) Add the magnetomotive forces found under (*c*) to get the total magnetomotive force.

(*e*) The total required magnetomotive force is equal to $4\pi ZI$, where Z is the number of turns of wire in the magnetizing coil and I is the magnetizing current in abamperes. Then if Z is given the value of I may be found, or if I is given the value of Z may be found.

Example.—The magnetic circuit of a dynamo consists of wrought iron (in armature core and field magnet cores†), cast iron (in field magnet yoke and pole pieces), and air gap. The dimensions of each portion of the magnet are estimated as follows: Wrought iron portion 50 centimeters long and 120 square centimeters in sectional area, cast iron portion 40 centimeters long and 202 square centimeters in sectional area, and air portion is 2.5 centimeters long and 300 square centimeters in sectional area. How many ampere-turns are required to force 1,600,000 lines of magnetic flux through the circuit, ignoring the leakage of flux through the surrounding air from pole piece to pole piece?

(*a*) $\begin{cases} \text{The flux density in the wrought iron portion is } B = 13{,}330 \\ \text{The flux density in the cast iron portion is } B = 7{,}270 \\ \text{The flux density in the air portion is } B = 5{,}330. \end{cases}$

(*b*) $\begin{cases} \text{The value of } H \text{ in the wrought iron is } H = 14.8 \text{ gausses.} \\ \text{The value of } H \text{ in the cast iron is } H = 30.0 \text{ gausses.} \\ \text{The value of } H \text{ in the air gaps is } H = 5330 \text{ gausses.} \end{cases}$

* No account is here taken of what is called magnetic leakage.

† Portions of the field magnet structure on which the magnetizing coils are wound.

(c) The magnetomotive force required for the wrought iron portion is 740 gauss-centimeters or gilberts.
The magnetomotive force required for the cast iron portion is 1,200 gilberts.
The magnetomotive force required for the air gaps is 13,320 gilberts.

(d) The total required magnetomotive force is 15,260 gilberts.

(e) Dividing 15,260 by 4π gives $ZI = 1{,}215$ abampere-turns or 12,150 ampere-turns.

6. To calculate the magnetic flux produced around a given magnetic circuit by a specified magnetomotive force.—When the given magnetic circuit consists of different materials, as in the above example, this problem is solved by calculating the magnetomotive forces required to produce a series of assumed values of flux, these results are plotted as a curve, and the flux corresponding to the given magnetomotive force is taken from this plotted curve.

Magnetic reluctance.—The electrical engineer usually carries out a magnetic circuit calculation as follows: After finding the flux density B in any portion of the magnetic circuit, and taking the value of μ from the table, he calculates the quantity $\frac{1}{\mu}\cdot\frac{l}{q}$ where l is the length in centimeters and q is the sectional area in square centimeters of the portion of the magnetic circuit. This quantity is called the *magnetic reluctance* of the portion. Having thus determined the magnetic reluctances of the various portions of the circuit, the reluctance of the entire magnetic circuit is found exactly as the electrical resistance of an entire electrical circuit is found from the resistances of the various portions of the electrical circuit. Then the magnetic flux through the circuit is found by dividing the magnetomotive force by the total reluctance of the magnetic circuit. This method is the exact equivalent of the method which is outlined above.

PROBLEMS.

1. An iron rod 2 centimeters square and 20 centimeters long is magnetized to an intensity of 1,000 units pole per square centimeter section when it is placed in a region which, but for the action of the poles of the rod, would be a uniform field parallel to the rod and of an intensity of 102 gausses. Assuming the poles of the rod to be concentrated at its ends, calculate the net magnetizing field at the center of the rod.

2. A laminated iron core two feet long and $\frac{1}{2}$ inch in diameter is wound from end to end with a layer of wire containing 50 turns. Assuming that the permeability of the iron is constant and equal to 1,150, and ignoring the demagnetizing action of the free poles of the rod, calculate the inductance of the winding in henrys.

3. The intensity of the magnetic field in the air gap between the pole face and the armature core of a dynamo is 3,500 gausses and the field is at right angles to pole face and armature surface. The distance across the air gap is 3/8 inch. Find the magnetomotive force across the air gap in gilberts and in ampere-turns.

4. A slim rod 25 centimeters long is made into a link which passes through a coil of 50 turns of wire in which a current of 15 amperes is flowing. Find the average value along the rod of the component parallel to the rod of the magnetic field due to the coil. Express the result in gausses.

5. A transformer has a laminated soft-iron core of which the sectional area is 120 square centimeters and the mean length of the magnetic circuit formed by the core is 100 centimeters. How much current must flow through a winding of 500 turns of wire to produce a magnetic flux of 1,767,000 lines?

6. How much flux will be produced through the laminated iron core of the previous problem by a current of 8.2 amperes in the 500 turns of wire?

7. How much magnetic flux would be forced through the magnetic circuit which is specified in the example of Art. 5 by a magnetomotive force of 13,500 ampere-turns?

7. **Proposition.**—*The magnetomotive force along a path in a magnetic field is equal to the work per unit pole done by the magnetic field upon a magnet pole while the pole is made to travel along the path*; that is, the magnetomotive force along a path is equal to W/m, where W is the work done by the field upon a pole of strength m while the pole is made to travel along the path. That is:

$$F = \frac{W}{m} \tag{4}$$

in which F is the magnetomotive force along the path.

Equation (4) is usually taken as the definition of magnetomotive force, but it is necessary here to derive it from the definition of magnetomotive force which is given in Art. 3. If H is the average value along a path of the component of a magnetic field parallel to the path at each point, then mH is the average force parallel to the path with which the field acts on a pole of strength m while the pole moves along the path so that $lmH = W$ is the work done on the pole by the field. Therefore $lH = \frac{W}{m}$ where lH is the magnetomotive force along the path.

8. **Proposition.**—The magnetomotive force of a coil is given by the equation:

$$F = 4\pi Zi \tag{3}$$

in which Z is the number of turns of wire in the coil, and i is the strength of the current in the wire in abamperes.

Proof.—Before proceeding to the derivation of equation (3), it is necessary to find an expression for the total work W done in keeping the current i in a coil constant while the magnetic flux through the opening of the coil is increased by a specified amount Φ; W being expressed in ergs, i in abamperes, and Φ in maxwells or lines. Now, while the flux is increasing, a back electromotive force equal to $Z\frac{d\Phi}{dt}$ abvolts is induced in the coil, and therefore (assuming the coil to have zero resistance for the

sake of simplicity of statement) an electromotive force $e = Z\frac{d\Phi}{dt}$ will have to act on the coil to keep the back electromotive force from decreasing the current. Consequently work will have to be done on the coil at the rate ei or $Z\frac{d\Phi}{dt} \times i$, to keep the current from decreasing. That is:

$$\frac{dW}{dt} = Zi \cdot \frac{d\Phi}{dt} \tag{i}$$

But Zi is a constant, because i is being kept from changing. Therefore by integrating (i) from $W = 0$ and $\Phi = 0$ we get:

$$W = Zi\Phi \tag{ii}$$

in which W is the work in ergs done to keep a current of i abamperes constant in a coil (of zero resistance) while the magnetic flux through the opening of the coil increases by the amount of Φ maxwells, and Z is the number of turns of wire in the coil.

We shall now proceed to the proof of equation (3). Let ZZ, Fig. 2, represent a coil of Z turns of wire. Imagine NS to be a *flexible* magnet. Let the north pole of this flexible magnet be carried through the coil and around to its initial position along the dotted path. The flexible magnet will then link with the coil of wire as shown in Fig. 3. Let F be the magnetomotive force along the dotted path, and let m be the strength of the pole N which has been carried around the path. Then, according to equation (4), Fm is the work done on the pole by the magnetic field of the coil. *This work done on the moving pole by the magnetic field of the coil is the work which*

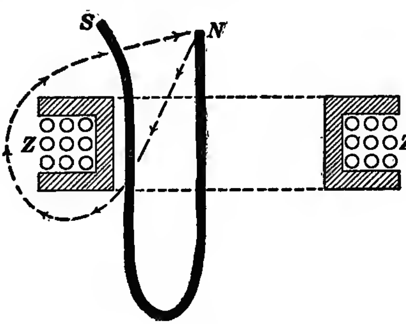

Fig. 2.

is spent in keeping the current constant in spite of the electromotive force induced in the coil by the moving pole. This is evident when we consider that work done on the pole by the field must be made good that is, energy must be supplied from somewhere: and when we consider that the energy stored in the system is the same in Figs. 2 and 3. Therefore, the only possible source of the energy for doing work on the moving pole is the work that is done to keep the current in the coil constant.

Now the two poles of the flexible magnet are in the same positions before and after the movement; therefore, of the total number of lines of force which *radiate* from these poles, the same number pass through the coil before and after the movement. On the other hand, the flux $4\pi m$ (see Art. 12 of chapter I), which passes along the magnet from pole to pole, passes through the coil after the movement, so that the flux through the opening of the coil is increased by the amount $4\pi m$ by the movement of the pole. Therefore, according to equation (ii), $Zi \times 4\pi m$ is the work spent in keeping the current constant during the movement of the pole and, since this is equal to the work Fm done upon the pole by the magnetic field of the coil, we have

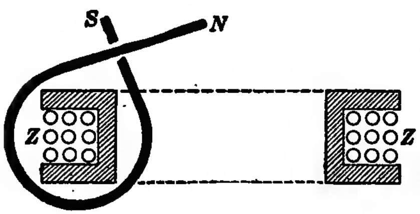

Fig. 3.

$$Fm = 4\pi i Zm$$

or

$$F = 4\pi Zi$$

9. Work required to magnetize iron.—When an iron rod is magnetized by sending an electric current through a coil of wire surrounding the rod, an opposing electromotive force is induced in the coil by the growing magnetism of the rod, and *the work done in forcing the current against this opposing electromotive force is the work expended in magnetizing the rod.*

The work W, in ergs, which is done in magnetizing V cubic centimeters of iron from a given initial flux density B' to a given final flux density B'', is given by the equation:

$$W = \frac{V}{4\pi}\int_{B'}^{B''} H \cdot dB \tag{5}$$

Proof.—In order to avoid the complications which arise on account of the perceptible demagnetizing action of the poles of a short iron rod, let us consider a very long slim rod l centimeters in length and q square centimeters in sectional area. Suppose this rod to be placed in a long coil having z turns of wire per centimeter of length or lz total turns. When the coil of wire is first connected to the battery or other source of current, the current in the coil (beginning at zero) rises in value during the time that the rod is being magnetized, and during this time the magnetic flux through the rod is growing in value. Let $d\Phi/dt$ be the rate at which the flux is increasing at a given instant, and let i be the value of the current at this instant. Then $lz \times d\Phi/dt$ is the induced electromotive force in the coil which at the given instant is opposing the current i, so that $lz \times d\Phi/dt \times i$ is the rate, dW/dt, at which work is being done at the given instant in magnetizing the rod. That is:

$$\frac{dW}{dt} = lzi \cdot \frac{d\Phi}{dt}$$

so that:

$$dW = lzi \cdot d\Phi \tag{i}$$

in which dW is the amount of work done during the time that the flux has increased by the amount $d\Phi$ and while the current has the mean value i.

Now $\Phi = Bq$ or $d\Phi = q \cdot dB$, and $zi = H/4\pi$ from equation (1) of Art. 2 of this Appenidx, so that equation (i) becomes:

$$dW = \frac{lq}{4\pi} H \cdot dB$$

or, since $lq = V$, we have:

$$dW = \frac{V}{4\pi} H \cdot dB$$

or

$$W = \frac{V}{4\pi} \int_{B'}^{B''} H \cdot dB$$

In magnetizing a short iron rod, more work is done than is accounted for by equation (5). The additional work goes to establish the magnetic field in the neighborhood of the magnetic poles of the rod. Equation (5) expresses the work which is spent within the iron.

10. Graphical representation of work done in magnetizing iron.—Let the curve Opp', Fig. 4, be drawn so that the coordinates represent corresponding values of B and H for a given sample of iron. The branch Op represents the values of B and H when the iron is being magnetized for the first time, and the branch pp' represents the values of B and H when, after the iron has been magnetized up to the point p, the value of H is slowly reduced to zero. *The curve of B and H for decreasing values of H does not coincide with the curve for increasing values of H.* Now, as explained later, the total area Opa represents the *work done upon the iron* in magnetizing it up to the point p, and the area $pp'a$ represents the *work which is regained from the iron* when the magnetizing field drops slowly to zero. *The work regained is less than the work required to magnetize the iron.* The work which is lost is represented by the shaded area in Fig. 4.

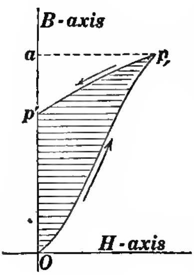

Fig. 4.

That area in Fig. 4 represents work may be shown as follows: Abscissas represent values of H to scale, so that we may write:

$$H = ax \tag{i}$$

Ordinates represent values of B to scale, so that we may write:

$$B = by \tag{ii}$$

or

$$dB = b \cdot dy$$

Substituting these values of H and dB in equation (5), we have

$$W = \frac{abV}{4\pi} \int x \cdot dy$$

in which a is the number of units of H represented by one unit of abscissa, and b is the number of units of B represented by one unit of ordinate in Fig. 4. Now $\int x \cdot dy$ is the area between any portion of the B and H curve and the y-axis. Therefore, $ab\,V/4\pi$ is the number of ergs of work represented by each unit of area between the B and H curve and the y-axis.

11. Magnetic hysteresis. The magnetic cycle.—The divergence of the B and H curve for increasing values of H from the B and H curve for decreasing values of H is called *magnetic hysteresis*; or, rather, the tendency of iron to retain a previous magnetic state which is the cause of this divergence is called magnetic hysteresis. One effect of magnet hysteresis is that the work regained when iron is demagnetized is less than the work which must be spent to magnetize the iron, as pointed out in Art. 10.

The magnetization of a given portion of the iron core of a transformer is reversed with each reversal of the alternating current which flows through the primary coil, the iron is thus repeatedly carried from a certain degree of magnetization in one direction (a certain positive value of B) to the same degree of magnetization in the opposite direction (the same negative value of B), and back to the original degree of magnetization. Such a magnetic double-reversal is called a *magnetic cycle.* At the end of a cycle the iron comes to precisely the same condition that it had at the beginning of the cycle. The importance of the

magnetic cycle in the determination of the amount of heat produced by magnetization will be appreciated from the following two statements:

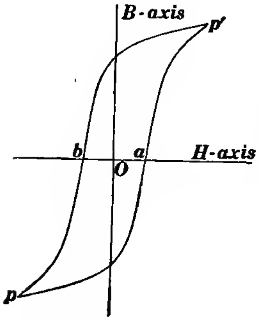

Fig. 5.

(*a*) When a mass of iron is magnetized along the B and H curve Op of Fig. 4 and then partially demagnetized along the curve pp', a portion of the work done upon the iron during the first stage Op is regained during the stage pp', a portion is lost as heat, and a portion remains in the iron as energy of magnetization; and no experimental method has been devised for determining the second and third porions of work separately.

(*b*) When, however, a mass of iron is carried through a magnetic cycle, the algebraic sum of the work spent upon the iron during the cycle is lost as heat, inasmuch as the magnetic energy in the iron is exactly the same at the beginning and at the end of the cycle.

Figure 5 shows the relation between B and H during a complete magnetic cycle. The total work spent on the iron is given by the value of the integral

$$\frac{abV}{4\pi}\int x \cdot dy \tag{i}$$

extended over the whole cycle; but the value of $\int x \cdot dy$ extended over the whole cycle is the area enclosed by the B and H curve. Therefore, the total energy, in ergs, lost in V cubic centimeters of iron per magnetic cycle, is equal to $ab\,V/4\pi \times$ area enclosed by the B and H curve. This energy loss is called the *hysteresis loss*, and it is all converted into heat. The meanings of a and b are explained in Art. 10.

The hysteresis loss per cycle increases with the range of flux

density, and it may be expressed with sufficient accuracy for most practical purposes by the empirical equation:

$$W = \eta V B^{1.6} \tag{6}$$

which is due to Steinmetz. In this equation W is the loss of energy in ergs per cycle, V is the volume of the iron in cubic centimeters, $\pm B$ is the range of flux density during the cycle, and η is a coefficient which is nearly constant for a given kind of iron or steel. The following table gives the approximate values of η for different kinds of iron and steel.

TABLE.

Values of Hysteretic Coefficient η.

Best quality of sheet iron for transformer cores, annealed	0.0015
Sheet iron for armature cores, annealed	0.003
Ordinary sheet iron, annealed	0.004
Soft annealed cast iron	0.008
Soft machine steel	0.0095
Cast steel	0.12
Hardened steel	0.25

12. The magnetic testing of iron (Rowlands' method). A ring-shaped sample of the iron or steel to be tested is wound with a magnetizing winding of Z turns of wire as indicated in Fig. 6, arrangements are made whereby the magnetizing current in this winding can be increased and decreased by sudden *steps*, an auxiliary winding of Z' turns is wound on the ring and connected to a ballistic galvanometer, and the throw of the ballistic is observed at every step of the magnetizing current. The object of the test is to get data from which a hysteresis loop like Fig. 5 can be plotted, but we will here explain only how to get the values of B and H for the extreme points of the hysteresis loop.

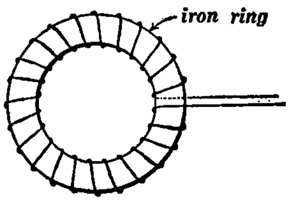

Fig. 6.

A measured magnetizing current of I abamperes is sent through the magnetizing winding and reversed several times so

as to wipe out residual effects of previous magnetic conditions of the sample, then a reversal of I changes the magnetic flux (through the sample) from a certain positive value Φ to an equal negative value Φ thus giving a total change of flux 2Φ which may be calculated from the observed throw of the ballistic galvanometer as explained in Art. 71 of Chapter V. Thus we get the value of Φ, and dividing Φ by the sectional area of the ring-shaped sample we get the value of flux density B. The corresponding value of H is equal to $4\pi ZI/l$ where I is the magnetizing current in abamperes and l is the mean circumferential length of the ring-shaped sample.

Problems.

8. (*a*) Find the work in ergs spent in magnetizing a wrought iron bar 3 inches square and 20 inches long from a neutral condition to $B = 16{,}000$ lines per square centimeter, using the tabular values of B and H given in Art. 2. (*b*) Find the work in ergs required to magnetize a cast-iron rod of the same size from a neutral condition to $B = 9{,}000$ lines per square centimeter.

Note.—Plot the B and H curves from the tabular values given in Art. 2. Divide the areas between the curves and the B axis into a number of narrow strips, and calculate the area of each strip. Add these areas to get the total area.

9. A transformer core contains 96 cubic inches of the best quality of transformer iron. The core is carried through 60 magnetic cycles per second between the limits $B = \pm 3{,}500$ gausses, by means of an alternating current. Find the hysteresis loss in the core in watts.

APPENDIX B.

ELEMENTARY THEORY OF ALTERNATING CURRENTS.

13. **Undamped oscillations and damped oscillations.**—An oscillating electrical circuit like Fig. 89 or Fig. 93 of Chapter V always has more or less resistance, and an oscillating circuit always loses energy more or less rapidly by the emission of electromagnetic waves. Therefore electrical oscillations once started die away rapidly. Such oscillations are called *damped oscillations.* The frequency equation (36) of Art. 78, Chapter V, applies to an ideal oscillating circuit which has no resistance and which loses no energy by the emission of electromagnetic waves, a circuit which once started would continue to oscillate indefinitely performs what would be called *undamped* oscillations.

When an oscillating circuit like Fig. 93 of Art. 78, Chapter V, has resistance R, equation (36) is not applicable, but the frequency is given by

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \qquad \text{(i)}$$

and each successive maximum positive value of the current is equal to the previous maximum value divided by ϵ^{aT}, where ϵ is the Naperian base, T is $1/f$ of a second, and

$$\alpha = \frac{R}{2L} \qquad \text{(ii)}$$

The current is called a *decaying oscillatory current,* and equations (i) and (ii) come from the solution of the differential equation of the circuit.

Consider a circuit like Fig. 93 of Chapter V, the capacity of the condenser being C, the inductance of the circuit being L and the resistance of the circuit being R. Let q be the charge on the condenser at any instant. Then the electromotive force across the condenser is q/C, this electromotive force pushes back

wards on the circuit so that it is to be considered as negative, part of it is used to overcome resistance and part of it is used to make the current charge. The part used to overcome resistance is Ri or $R\frac{dq}{dt}$, and the part used to make the current change is $L\frac{di}{dt}$ or $L\frac{d^2q}{dt^2}$. Therefore we have

$$\frac{-q}{C} = R\frac{dq}{dt} + L\frac{d^2q}{dt^2}$$

whence we have

$$q + CR\frac{dq}{dt} + CL\frac{d^2q}{dt^2} = 0$$

This is the differential equation of the circuit, and its general solution is

$$q = Q\epsilon^{-at} \sin(\omega t - \theta)$$

or, since $i = \frac{dq}{dt}$, we have

$$i = I\epsilon^{-at} \cos(\omega t - \theta) \tag{iii}$$

where I is written for the value of i when $t = 0$. This is the equation of the decaying oscillatory current.

14. **Free and forced oscillations.**—When a pendulum is started and left to itself its oscillations are called its *free oscillations*; if, however, a periodic force (a steady series of impulses) acts on a pendulum the pendulum soon settles to a steady state of oscillatory motion in which the frequency is exactly the same as the frequency of the force, regardless of the frequency of the free oscillations of the pendulum. This steady state of oscillation which is produced by a periodic force constitutes what is called *forced oscillations*. Similarly, an electric circuit like Fig. 93 of Chapter V performs *free oscillations* of which the frequency is given by equation (i) above when it is started and left to itself, but if such a circuit (or any circuit) is acted upon by an alternating electromotive force of any given frequency it soon settles to a steady condition of oscillation (*forced oscillation*) in which the frequency is that of the applied alternating electromotive force

regardless of the frequency of the free oscillations of the circuit.

The whole of the elementary theory of alternating currents has to do with steady states of forced oscillation.

15. The alternating-current ammeter and voltmeter.—In using an ordinary alternating-current ammeter or voltmeter the frequency is so high that the pointer of the instrument stands still at a definite scale reading, so many "amperes" or "volts" as the case may be, and it is important to consider precisely what this constant reading means. It evidently does not mean the average value of the alternating current or electromotive force, because the average value is zero. Indeed a direct-current ammeter or voltmeter of the type described in Art. 18 of Chapter I measures the true average value of a rapidly fluctuating current or electromotive force, and such an instrument gives *zero reading* for an alternating current or electromotive force.

To understand the meaning of an *ampere reading* or *voltage reading* on an alternating-current instrument let us consider the electrodynamometer type of ammeter or voltmeter. This instrument consists of a fixed coil and a delicately pivoted coil connected in series with each other, the current which produces the deflection flows through both coils, the fixed coil exerts a force on the pivoted coil which moves the coil and the attached pointer, *and this force is at each instant strictly proportional to the square of the current* for a given position of the pivoted coil.

(*a*) Let $\mathcal{I}$ be the constant or direct current which produces a certain deflection or scale reading s on an electrodynamometer type of ammeter. Then the steady deflecting force exerted on the pivoted coil is equal to $k\ ^2$, where k is a constant.

(*b*) Let i be the varying value of an alternating current which gives the same steady deflection s. Then the instantaneous value of the force exerted on the pivoted coil is ki^2, and the average value of this force is $k \times$ average i^2.

Now the steady deflecting force under (*a*) must be equal to

the average deflecting force under (*b*) because the deflection s is the same. Therefore*

$$k\mathcal{I}^2 = k \times \text{average } i^2$$

or

$$\mathcal{I}^2 = \text{average } i^2$$

or

$$\mathcal{I} = \sqrt{\text{average } i^2} \tag{i}$$

But the value of $\mathcal{I}$ is marked on the ammeter scale where the pointer stands under (*a*) above. Therefore $\mathcal{I}$ is the reading s of the ammeter under (*b*). *Therefore, according to equation* (i), *the reading of an alternating-current ammeter gives the square-root-of-the-average-value-of-the-square of the alternating current.*

The voltmeter.—A large non-inductive† resistance is connected in series with the fixed and pivoted coils of an electrodynamometer instrument so that the instrument may be connected directly across the supply mains. Let R be the value of this resistance including the resistance of the coils of the instrument.

(*a*) Let $\mathcal{E}$ be the constant or direct electromotive force which produces a certain deflection or scale reading s. Then the steady deflecting force exerted on the pivoted coil is $k\left(\frac{\mathcal{E}}{R}\right)^2$.

(*b*) Let e be the varying value of an alternating electromotive force which gives the same steady deflection s. Then‡ the instantaneous value of the force exerted on the pivoted coil is $k\left(\frac{e}{R}\right)^2$, and the average value of this force is $\frac{k}{R^2} \times \text{average } e^2$.

* The reader should use his calculus to establish the following two propositions, namely, (1) Average $(ax) = a \times$ average x, where a is a constant, and (2) Average $(x + y) =$ average $x +$ average y.

† A circuit is said to be non-inductive when its inductance L is negligibly small, and the inductance is negligibly small when the electromotive force $L\frac{di}{dt}$ which is required to make the current change is very small as compared with the electromotive force Ri which is required to overcome resistance.

‡ The electromotive force e which acts on a circuit of resistance R and inductance L is equal to $Ri + L\frac{di}{dt}$, and if $L\frac{dt}{di}$ is negligible we have $e = Ri$ or $i = e/R$.

Now the steady deflecting force under (*a*) must be equal to the average deflecting force under (*b*) because the deflection s is the same. Therefore

$$k\left(\frac{\mathcal{E}}{R}\right)^2 = \frac{k}{R^2} \times \text{average } e^2$$

or

$$\mathcal{E}^2 = \text{average } e^2$$

or

$$\mathcal{E} = \sqrt{\text{average } e^2} \tag{ii}$$

But the value of $\mathcal{E}$ is marked on the voltmeter scale where the pointer stands under (*a*) above. Therefore $\mathcal{E}$ is the reading s of the voltmeter under (*b*) above. *Therefore, according to equation* (ii), *the reading of an alternating voltmeter gives the square-root-of-the-average-value-of-the-square of the alternating electromotive force.*

Effective values of alternating electromotive force and current.—The value of an alternating electromotive force as measured by a voltmeter, the square-root-of-the-average-value-of-e^2, is called the *effective value* or *voltmeter value* of the electromotive force, and it is represented by the letter E. The value of an alternating current as measured by an ammeter, the square-root-of-the-average-value-of-i^2, is called the *effective value* or *ammeter value* of the current and it is represented by the letter I.

Heating effect of an alternating current.—Let i be the value of an alternating current at a given instant, and let R be the resistance of the circuit or coil through which the current flows. Then Ri^2 watts is the rate at which heat is generated in the circuit or coil at the given instant, and $R \times$ average i^2 is the average rate of generation of heat in the circuit or coil. But $I = \sqrt{\text{average } i^2}$ so that $I^2 =$ average i^2. Therefore $R \times$ average $i^2 = RI^2$ or *the average rate of generation of heat in a coil or circuit is found by multiplying the resistance by the square of the effective or ammeter-value of the current.*

PROBLEM.

10. A device is arranged to connect a voltmeter to direct-current supply mains and disconnect it repeatedly, connection never reversed. The voltmeter is connected for 0.01 second and disconnected for 0.04 second, over and over. The supply voltage is 110 volts. Find the reading of the voltmeter (*a*) When it is of the direct-current type, and (*b*) When it is of the alternating-current type.

16. Harmonic alternating electromotive force or current.— In many cases the curve which represents the varying value of an alternating electromotive force or current is a curve of sines as in Fig. 7. In such a case the electromotive force or current

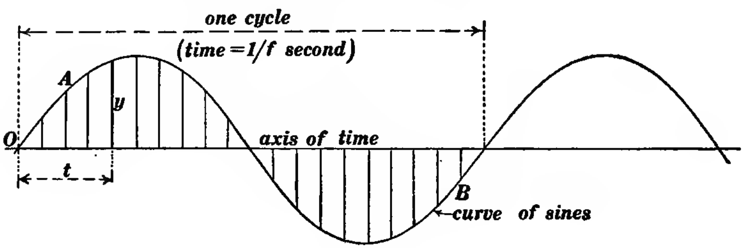

Fig. 7.

is called a *harmonic alternating electromotive force or current*, or simply, a *harmonic electromotive force or current.* Let f be the frequency (number of cycles per second), then $1/f$ is the so-called period (number of seconds or the fraction of a second per cycle) as indicated in the figure. The abscissas of a curve of sines are most conveniently thought of as angles, and, of course, the angle corresponding to a whole cycle is 360° or 2π radians. Therefore the angle α corresponding to any given time t, reckoned from the point O in Fig. 7, is derived from the proportion $\alpha : 2\pi :: t : 1/f$. Therefore $\alpha = 2\pi f \cdot t$ and the equation to the sine curve in Fig. 7 is

$$e = \mathrm{E} \sin 2\pi f \cdot t \tag{i}$$

where $\mathbf{E}$ is the maximum ordinate (volts) and e is the value of the alternating electromotive force at any instant t; or

$$i = \mathbf{I} \sin 2\pi f \cdot t \tag{ii}$$

where $\mathbf{I}$ is the maximum ordinate (amperes) and i is the value of the alternating current at any instant t. It is customary to represent the factor $2\pi f$ by the Greek letter ω. That is

$$\omega = 2\pi f \tag{7}$$

so that equations (i) and (ii) become

$$e = \mathbf{E} \sin \omega t \tag{8}$$

and

$$i = \mathbf{I} \sin \omega t \tag{9}$$

17. The clock diagram.—Imagine the line OP in Fig. 8 to make f revolutions per second or $2\pi f (= \omega)$ radians per second about the point O in the direction of the curved arrow, and let time be reckoned from the instant that OP coincides with the dotted line OX. Then at time t the line will be in the position shown, and Ob will be equal to OP times the sine of ωt. Therefore:—

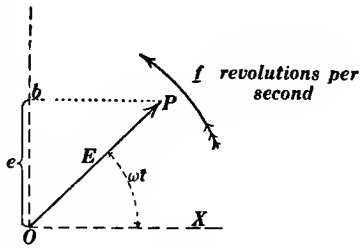

Fig. 8

If the length of the rotating line OP represents the maximum value $\mathbf{E}$ of a harmonic alternating electromotive force, the projection Ob will at each instant represent the value e of the alternating electromotive force; or

If the length of the rotating line OP represents the maximum value $\mathbf{I}$ of a harmonic alternating current, the projection Ob will at each instant represent the value i of the alternating current.

18. Phase difference.—Consider a harmonic alternating electromotive force e and a harmonic alternating current i of the

same frequency. It often happens that the current reaches its maximum value *after* the electromotive vorce has passed its maximum value. In this case the current is said to *lag behind* the electromotive force in phase. In some cases the current reaches its maximum value *before* the electromotive force reaches its maximum value; and in such a case the current is said to be *ahead of* the electromotive force in phase. When a current is behind an electromotive force in phase, as indicated in Fig. 9,

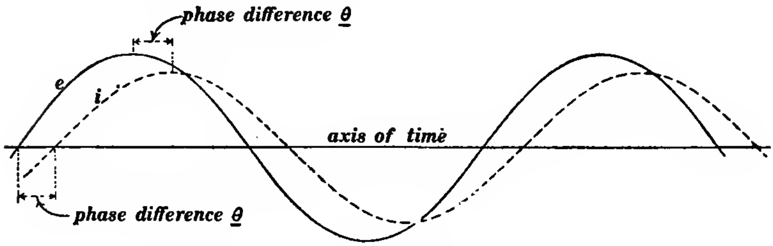

Fig. 9.

Current i behind electromotive force e in phase.

it is called a *lagging current*; when a current is ahead of an electromotive force in phase, as indicated in Fig. 10, it is called a *leading current*.

The phase difference in Figs. 9 and 10 is, of course, a definite but very small time interval, a definite fraction of a cycle, and it corresponds also to a definite angle θ (a definite fraction of 2π radians). The current in Fig. 9 is θ radians behind the electro-

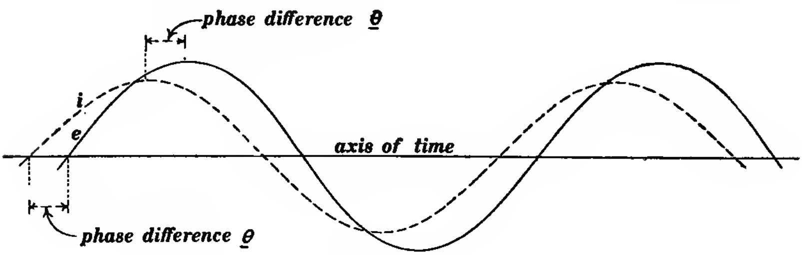

Fig. 10.

Current i ahead of electromotive force e in phase.

motive force in phase, and in Fig. 10 the current is θ radians ahead of the electromotive force in phase.

Phase difference in the clock diagram.—Figure 11 is a clock diagram corresponding exactly to Fig. 9, the alternating current which is represented by the varying projection of the rotating line **I** is θ radians behind the electromotive force which is represented by the varying projection of the rotating line **E**; and Fig. 12 is a clock diagram corrseponding exactly to Fig. 10.

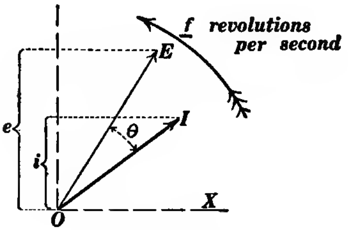

Fig. 11.

Corresponding to Fig. 9. Current behind voltage in phase.

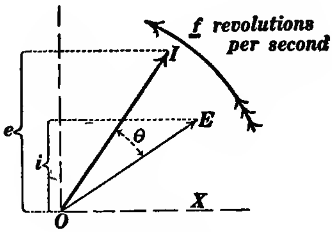

Fig. 12.

Corresponding to Fig. 10. Current ahead of voltage in phase

If we reckon time from the instant that the line **E** coincides with OX, then the angle **E**OX will be $2\pi ft$ or ωt radians, the angle **I**OX in Fig. 11 is $\omega t - \theta$, and the angle **I**OX in Fig. 12 is $\omega t + \theta$. Therefore we have

$$\left.\begin{array}{lll} \text{a given harmonic electromotive force} & e = \mathbf{E} \sin \omega t & \\ \text{a current lagging } \theta^\circ \text{ behind } e & i = \mathbf{I} \sin (\omega t - \theta) & \\ \text{a current leading } \theta^\circ \text{ ahead of } e & i = \mathbf{I} \sin (\omega t + \theta) & \end{array}\right\} \quad (10)$$

Remark.—Consider a rotating clock-diagram vector whose varying projection represents a harmonic alternating electromotive force or current. We shall hereafter speak of such a vector simply as *representing* the electromotive force or current.

19. Relation between maximum value and effective value of harmonic alternating electromotive force or current.—The effective value of an alternating electromotive force or current, as measured by a voltmeter or ammeter, is the square-root-of-the-

average-value-of-the-square of the electromotive force or current as explained in Art. 15; and the maximum value of a harmonic alternating electromotive force or current is the quantity $\mathbf{E}$ in equation (8) or the quantity $\mathbf{I}$ in equation (9), as explained in Art. 16.

The effective value of a harmonic alternating electromotive force (or current) is equal to its maximum value $\mathbf{E}$ *(or* $\mathbf{I}$*) divided by the square root of* 2. That is

$$E = \frac{\mathbf{E}}{\sqrt{2}} \tag{11}$$

$$I = \frac{\mathbf{I}}{\sqrt{2}} \tag{12}$$

in which E and I represent effective values as indicated by a voltmeter or ammeter. These relations may be established by reducing the expression, (effective value)^2 $= \frac{1}{2\pi}\int_0^{2\pi} \mathbf{E}^2 \sin^2 x \cdot dx$.

Use of effective values instead of maximum values in the clock diagram.—According to Art. 17 the lengths of the vectors or lines in a clock diagram represent maximum values $\mathbf{E}$ and $\mathbf{I}$; but the ratio of maximum values to effective values is constant according to equations (11) and (12), *therefore it is permissible to think of the clock-diagram vectors as representing effective values.* Indeed this is always done when the clock-diagram is used to show the relation between several alternating electromotive forces or currents.

PROBLEMS.

11. A 60-cycle harmonic alternating current is 25° behind an electromotive force in phase. What is the value of the phase difference expressed as a fraction of a cycle? What is the value of the phase difference expressed as a fraction of a second?

12. The voltmeter value of a harmonic electromotive force is 110 volts. What is its maximum *value*? This alternating electromotive force produces harmonic alternating current of 20 amperes (ammeter value) through a circuit and the current

is in phase with the electromotive force. What is the maximum rate at which work is being delivered to the circuit?

20. Addition of harmonic alternating electromotive forces of the same frequency.—Consider two alternators A and B connected in series with each other as shown in Fig. 13a. Let a and b be the values of the electromotive forces of machines A and B respectively at any instant, and let c be the value

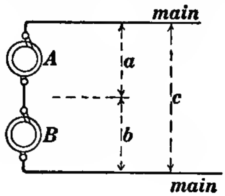

Fig. 13a.

a, b and c represent instantaneous values of electromotive force and $c = a + b$.

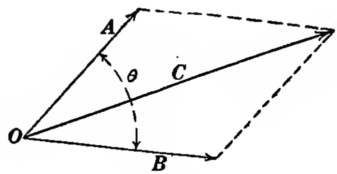

Fig. 13b.

Lengths of A, B and C represent voltmeter values of a, b and c.

at the same instant of the electromotive force between the mains. Then $c = a + b$ at each instant. If a and b have the same frequency f (and a constant phase difference θ), *then* c *will be a simple harmonic electromotive force of frequency* f as represented by the clock-diagram vector C in Fig. 13b. To make this evident let us think of A, B and C as representing maximum values so that their projections will represent instantaneous values *but the projection of the diagonal* C *is at each instant equal to the sum of the projections of the sides* A *and* B *of the parallelogram.* Therefore $c = a + b$, where c, a and b are the projections of C, A and B.

Resolution of a harmonic alternating electromotive force into harmonic parts.—Any given harmonic alternating electromotive force c (represented by the clock-diagram vector C in Fig. 13b) may be resolved into two harmonic alternating electro-

motive forces a and b (represented by the clock-diagram vectors A and B in Fig. 13b).

21. Addition of harmonic alternating currents of the same frequency.—Consider two alternators A and B connected in parallel with each other as indicated in Fig. 14a. Let a and b

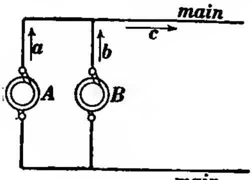

Fig. 14a.

a, b and c represent instantaneous values of current and $c = a + b$.

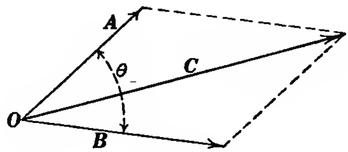

Fig. 14b.

Lengths of A, B and C represent ammeter values of a, b and c.

be the values of the currents delivered by machines A and B respectively at any instant and let c be the value at the same instant of the current in the mains. Then $c = a + b$ at each instant. If a and b have the same frequency f (and a constant phase difference θ), *then c will be a simple harmonic current of frequency f* as represented by the clock-diagram vector C in Fig. 14b.

Resolution of a harmonic alternating current into parts.—Any given harmonic alternating current c (represented by the clock-diagram vector C in Fig. 14b) may be resolved into two harmonic alternating currents a and b (represented by the clock-diagram vectors A and B in Fig. 14b).

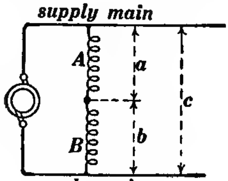

Fig. 15a.

a, b and c represent instantaneous values of electromotive force and $c = a + b$

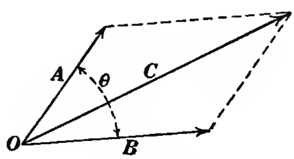

Fig. 15b.

Lengths of A, B and C represent voltmeter values of a, b and c.

22. Examples of resolution into parts.—(*a*) *Dividing of an alternating electromotive force between two coils in series.* Figure 15*a* shows two coils, or units of any kind, connected in series between alternating-current supply mains. The supply voltage c divides into two parts a and b such that $c = a + b$ at each instant; but from Fig. 15*b* it is evident that the voltmeter value of c is *not* equal to the sum of the voltmeter values of a and b, unless the phase difference θ is zero.

(*b*) *Dividing of an alternating current between two coils in parallel.* Figure 16*a* shows two coils, or units of any kind, connected in parallel with each other between alternating-current supply mains. The total current c divides into two parts a and b such that $c = a + b$ at each instant; but from Fig. 16*b* it is evident that the ammeter value of c is *not* equal to the sum of the ammeter values of a and b, unless the phase difference θ is zero.

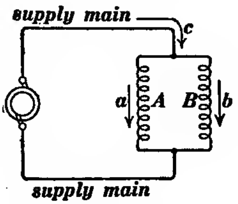

Fig. 16*a*.

a, b and c represent instantaneous values of current and $c = a + b$.

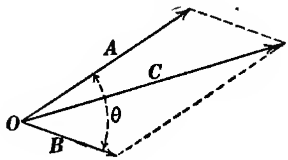

Fig. 16*b*.

Lengths of A, B and C represent ammeter values of a, b and c.

PROBLEMS.

Electromotive forces and currents understood to be harmonic.

13. Two similar alternators running at precisely the same speed are connected in parallel as indicated in Fig. 14*a*. The current A is 60 amperes, the current B is 40 amperes, and the current C is 85 amperes (ammeter values, of course). What is the phase difference of currents A and B?

14. The voltages A, B and C in Fig. 15*a*, as read by a voltmeter are found to be 75 volts, 80 volts and 110 volts respectively. What is the phase difference between A and B?

15. The currents A, B and C in Fig. 16*a*, as read by an ammeter are found to be 10.3 amperes 9.9 amperes and 1.1 amperes respectively (current C being very much smaller than either A or B). What is the phase difference between A and B?

Note.—The electromotive force C in Fig. 15*a* may be much smaller than either A or B.

23. Instantaneous and average values of power.—Let e be the value at a given instant of the electromotive force acting on a receiving circuit, and let i be the value of the current at the same instant; then ei is the power in watts which is being delivered to the receiving circuit at the given instant, and the average value of ei is the average power delivered to the receiving circuit. This *average power* is usually called, simply, the *power* delivered to the receiving circuit, and it is ordinarily *not* equal to the product EI where E is the voltmeter value of the electromotive force and I is the ammeter value of the current.

Some idea of the pulsating character of the instantaneous power ei as delivered by alternating-current supply mains may

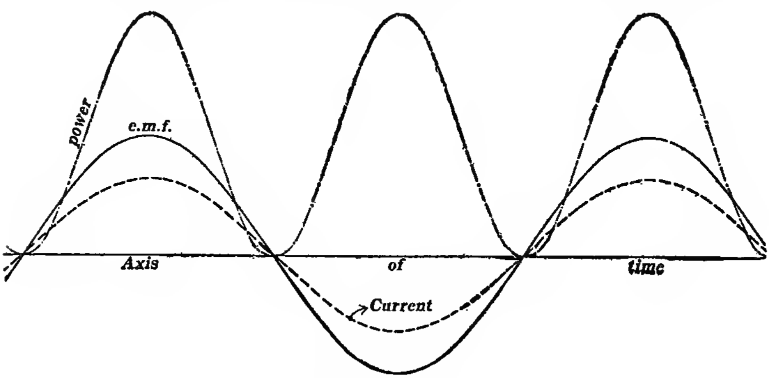

Fig. 17.

Electromotive force e and current i in phase with each other. Average power in this particular case is equal to EI.

be obtained from Figs. 17, 18 and 19. Negative values of the power ei mean power returned to the supply mains by the receiving circuit.

The relation between electromotive force e, current i, and power ei in Fig. 19 is completely analogous to the relation be-

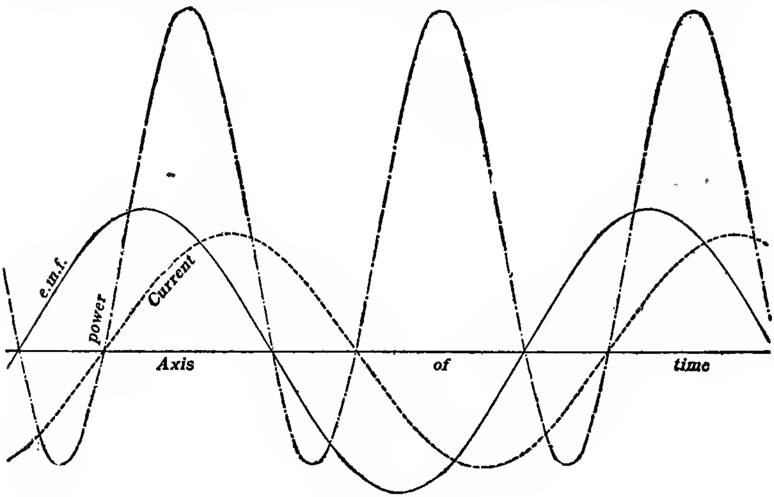

Fig. 18.

Current i is 60° (one sixth of a cycle) behind electromotive force e in phase. Average power in this particular case is $\frac{1}{2}EI$.

tween the torque e which is exerted by the hair spring on the balance wheel of a watch, the angular velocity i of the wheel,

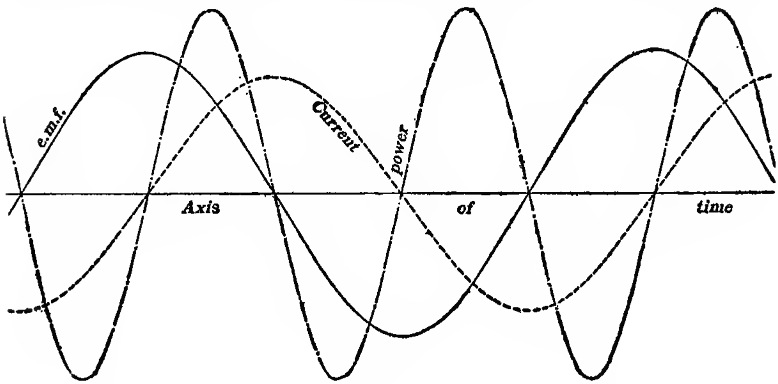

Fig. 19.

Current i is 90° (one quarter of a cycle) behind electromotive force e in phase. Average power in this particular case is zero.

and the power ei or rate at which work is delivered to the balance wheel by the spring. Consider the instant when the balance wheel is at its extreme position; during the following quarter of a cycle the torque e is setting the wheel in motion, e and i are both in the same direction, and ei is positive which means that work is done by the spring on the wheel; during the next quarter of a cycle the wheel is being stopped by the spring, e is opposed to i, and ei is negative which means that work is being done on the spring by the wheel; and so on. In this statement friction is ignored.

24. **The wattmeter.**—Power taken from alternating-current supply mains is nearly always measured by means of a *wattmeter* which consists of a delicately pivoted coil of fine wire A connected directly across the supply mains in series with a large non-inductive resistance R, and a stationary coil of course wire B connected in series with the receiving circuit L as indicated in Fig. 20. The force action between the coils A and B moves the pivoted coil A, and the pointer which is attached to the pivoted coil plays over a divided scale.

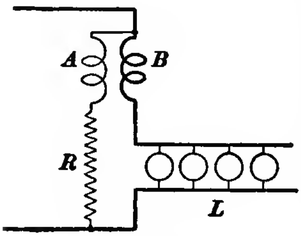

Fig. 20.

(*a*) Let s be the scale reading of the instrument when the instrument is connected to direct-current supply mains (connections the same as in Fig. 20) with constant voltage $\mathcal{E}$ across the mains and with constant current $\mathcal{I}$ flowing through coil B and the receiving circuit. The force exerted on the pivoted coil A is then equal to $k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$, where k is a constant.

(*b*) Let the instrument give the same scale reading s when it is connected to alternating-current supply mains as indicated in Fig. 20, let e be the value of the alternating electromotive force at a given instant, and let i be the value at the same instant of the alternating current flowing through coil B and

the receiving circuit. Then the value of the current in coil A at the given instant is e/R, the instantaneous value of the force exerted on the pivoted coil is $k \cdot \frac{e}{R} \cdot i$, and the average value of this force is $\frac{k}{R} \times$ average ei. But since the deflection of the instrument is the same as under (*a*) above this average force must be equal to the steady force $k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$. Therefore

$$\frac{k}{R} \times \text{average } ei = k \cdot \frac{\mathcal{E}}{R} \cdot \mathcal{I}$$

or

$$\text{average } ei = \mathcal{E}\mathcal{I}$$

But average ei is the power delivered by the alternating current mains, and $\mathcal{E}\mathcal{I}$ is the wattmeter reading the (steady direct-current power which gives the same deflection). Therefore *the wattmeter indicates alternating-current power correctly when it has been calibrated to read direct-current power, the circuit AR being noninductive.*

25. **Power factor of a receiving circuit.**—Let E be the voltmeter value of the alternating electromotive force between the supply mains, let I be the ammeter value of the alternating current which is delivered to a receiving circuit, and let P be the power delivered to the receiving circuit as measured by a wattmeter. Then EI is usually larger in value than P, and the quotient P/EI is called the *power factor* p of the receiving circuit. That is

$$p = \frac{P}{EI} \tag{13}$$

The power factor of a non-inductive circuit is unity.

26. **Expression for average power P in case of harmonic alternating electromotive force and current.**—The discussion in Arts. 23 to 25 is not restricted to harmonic electromotive force and current, it is entirely general. When, however,

electromotive force and current are harmonic, the average power P (the average value of ei) is

$$P = EI \cos \theta \tag{14}$$

where E is the voltmeter value of the electromotive force which acts on the receiving circuit, I is the ammeter value of the current delivered to the receiving circuit, and θ is the phase difference between electromotive force and current.

By comparing equation (14) with equation (13) it may be seen that in case of harmonic electromotive force and current the power factor of a receiving circuit is equal to $\cos \theta$ where θ is the phase difference between electromotive force and current.

Equation (14) may be established by reducing the expression

$$P = \text{average } ei = \frac{\omega}{2\pi} \int_{\omega t=0}^{\omega t=2\pi} \mathbf{E} \sin \omega t \, \mathbf{I} \sin (\omega t \pm \theta) \cdot dt$$

Suppose a cook were to stir pan-cake batter by the rapid to and fro motion of a light spoon; only a small part of the force exerted on the spoon would be used to accelerate and decelerate the spoon; the greater part of the force would be used to overcome the resistance of the batter and do useful work. Suppose, however, that a cook should attempt to stir pan-cake batter by means of a heavy coupling pin! Most of the force exerted on the pin would be used to produce acceleration and deceleration, and only a very small part of the force would be used to overcome the resistance of the batter and do useful work. The light spoon is the exact analog of a high-power-factor receiving circuit, and the heavy coupling pin used as a spoon is the exact mechanical analog of a low-power-factor receiving circuit.

PROBLEMS.

16. A receiving circuit takes 50 amperes from 110-volt alternating-current supply mains, and the power delivered to the receiving circuit, as measured by a wattmeter is 4620 watts. What is the power factor of the receiving circuit? What is the

phase difference between electromotive force and current on the assumption that both are harmonic?

17. The electromotive force e in Fig. 17 is harmonic and its maximum value is 155.6 volts. The current i is harmonic and its maximum value is 141.4 amperes. Find (*a*) voltmeter value of e, (*b*) ammeter value of i, (*c*) average value of ei, and (*d*) maximum value of ei.

18. The electromotive force e in Fig. 18 is harmonic and its maximum value is 311.2 volts, and the current i is harmonic and its maximum value is 212.1 amperes. Find (*a*) voltmeter value of e, (*b*) ammeter value of i, (*c*) average value of ei.

19. Two alternators A and B are connected in series. The electromotive force of A is 1,100 volts, and the electromotive force of B is 1,200 volts. The electromotive force of A is 90° ahead of the electromotive force of B in phase. (*a*) What is their combined electromotive force? The two alternators give a current of 125 amperes which lags 30° behind their resultant electromotive force in phase. (*b*) What is the power output of each alternator?

20. The electromotive force of alternator A in the previous problem is 135° ahead of the electromotive force of alternator B in phase. A current of 120 amperes flows through both alternators, lagging 30° behind their resultant electromotive force. What is the power output of each alternator.

21. A non-inductive rheostat of which the resistance is 120 ohms is connected in series with an alternating-current fan motor to 220-volt alternating supply mains. The voltage across the 120-ohm rheostat is 125 volts, and the voltage across the fan motor is 128 volts. Find (*a*) the current flowing through the circuit, (*b*) the phase difference between the current and the voltage across the fan motor, and (*c*) the power delivered to the fan motor. Ans. (*a*) 1.04 amperes, (*b*) 59° 10′, (*c*) 68.34 watts.

Note —It is impossible from the data of this problem to tell whether the voltage across the 120-ohm rheostat is ahead of or behind the voltage across the fan motor in phase. As a matter of fact, the voltage across the 120-ohm rheostat is in phase

with the current, and the voltage across the fan motor is ahead of the current in phase.

22. An alternator delivers 200 amperes of current to glow lamps, and 75 amperes to start an induction motor. The power factor of the motor while starting is 0.3. Find the total current on the assumption that both currents are harmonic.

---

27. The voltage-current relation. Special cases.—The most important equation in the elementary theory of alternating currents is the equation expressing the harmonic electromotive force required to maintain a given harmonic alternating current

$$i = \mathrm{I} \sin \omega t \tag{i}$$

in a given circuit. To establish the voltage-current equation in its general form it is necessary to consider the following special cases.

(*a*) **Electromotive force required to maintain the given current in a non-inductive receiving circuit of resistance** R.—The required electromotive force, e, is equal to Ri at each instant. Therefore, using equation (i), we have

$$e = R\mathrm{I} \sin \omega t \tag{a}$$

This electromotive force is in phase with i, and its maximum value is equal to $R\mathrm{I}$, where I is the maximum value of the given current.

(*b*) **Electromotive force required to maintain the given harmonic current see equation** (i) **in a circuit of inductance** L **but of zero resistance.**—In this case the electromotive force is used only to make the current increase and decrease, and it is therefore at each instant given by the equation $e = L\frac{di}{dt}$, according to Art. 65 of Chapter IV. Therefore, differentiating equation (i), we get $\frac{di}{dt} = \omega \mathrm{I} \cos \omega t$ but $\cos \omega t = \sin(\omega t + 90°)$ so that

$\frac{di}{dt} = \omega\mathbf{I}\sin(\omega t + 90°)$ and therefore the required electromotive force is

$$e = \omega L\mathbf{I}\sin(\omega t + 90°) \tag{b}$$

that is to say, *the required electromotive force is* 90° *ahead of* i *in phase* [see equations (10) of this Appendix], *and its maximum value is* $\omega L\mathbf{I}$.

Take a heavy weight in the hands and move it rapidly to and fro: the maximum velocity **I** of the weight in the positive direction occurs one quarter of a cycle after the maximum positive pull **E** has been exerted on the weight. That is to say, the alternating force which is required to make the weight oscillate to and fro is 90° ahead of the velocity i in phase. This is the exact mechanical analog of case b which is discussed above.

(*c*) **Electromotive force required to maintain the given harmonic current "through" a condenser of which the capacity is** C **farads.** This problem is greatly simplified by first considering the electromotive force required to make the charge q on the condenser vary harmonically so that we may have:

$$q = \mathbf{Q}\sin\omega t \tag{ii}$$

The required electromotive force is at each instant equal to q/C according to equation (30) of Art. 71. Therefore the required electromotive force is

$$e = \frac{q}{C} = \frac{\mathbf{Q}}{C}\cdot\sin\omega t \tag{iii}$$

But the varying charge on the condenser involves a flow of current, namely, $i = \frac{dq}{dt}$. Therefore, differentiating equation (ii) we get $i = \frac{dq}{dt} = \omega\mathbf{Q}\cos\omega t = \omega\mathbf{Q}\sin(\omega t + 90°)$, so that

$$i = \omega\mathbf{Q}\sin(\omega t + 90°) \tag{iv}$$

Equations (iv) and (iii) evidently give us a solution of our original problem because i as expressed by equation (iv) may be thought

of as a *given* harmonic current, and e as given by equation (iii) is the electromotive force required to maintain the current i through the condenser. (*c*) *The required electromotive force is* 90° *behind* i *in phase and its maximum value is* $\mathbf{I}/\omega C$ *where* **I** *is the maximum value of* i. This is evident when we compare equation (iv) with equations (10) of this appendix and when we consider that the maximum value of e is $\mathbf{Q}/C$ and the maximum value of i is $\omega\mathbf{Q}$. It is desired however to get an expression for the electromotive force required to maintain a given current as expressed by equation (i), and from the statement (*c*) we get

$$e = \frac{\mathbf{I}}{\omega C} \sin\,(\omega t - 90^\circ) \qquad (c)$$

as the electromotive force required to maintian the current $i = \mathbf{I} \sin \omega t$ through a condenser of capacity C.

One end of a flat spring is clamped in a vise, and the other end is moved back and forth by the hand, bending the spring, of course. The maximum velocity **I** of the end of the spring in the positive direction occurs one quarter of a cycle before the maximum positive pull **E** is exerted on the end of the spring. That is to say, the alternating force which is required to make the end of the spring move back and forth is 90° behind the velocity i in phase. This is the exact mechanical analog of case c which is discussed above.

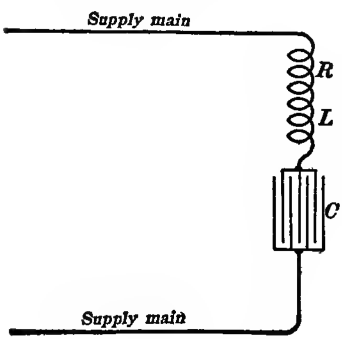

Fig. 21.

28. Voltage-current relation. General case.— *The electromotive force required to maintain a given harmonic current* [see equation (i)] *in a circuit of resistance* R *and inductance* L *and containing a condenser of capacity* C, *as shown in Fig. 21.* This electromotive force is the sum of the electromotive forces ex-

pressed by equations (a), (b) and (c) above. Therefore the required electromotive force is

$$e = R\mathbf{I} \sin \omega t + \omega L\mathbf{I} \sin (\omega t + 90°) + \frac{\mathbf{I}}{\omega C} \sin (\omega t - 90°) \qquad (d)$$

and this equation may be most easily reduced to its simplest form by using the clock diagram of Fig. 22 in which the three

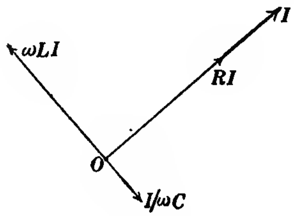

Fig. 22.

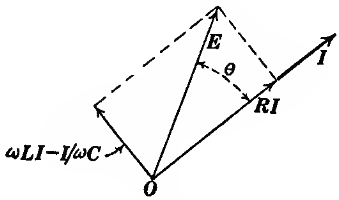

Fig. 23.

parts a, b and c of the required electromotive force are represented in value and in phase by the clock-diagram vectors $R\mathbf{I}$, $\omega L\mathbf{I}$ and $\mathbf{I}/\omega C$ respectively. The resultant of these three vectors is evidently the vector $\mathbf{E}$ in Fig. 23 from which we get

$$\mathbf{E}^2 = (R\mathbf{I})^2 + \left[\left(\omega L - \frac{1}{\omega C}\right)\mathbf{I}\right]^2$$

so that

$$\mathbf{E} = \mathbf{I}\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

whence, putting

$$X = \omega L - \frac{1}{\omega C} \qquad (15)$$

and, using equations (11) and (12) so as to introduce effective values E and I instead of maximum values $\mathbf{E}$ and $\mathbf{I}$, we get

$$\left.\begin{aligned} E &= I\sqrt{R^2 + X^2} \\ \text{or} \qquad I &= \frac{E}{\sqrt{R^2 + X^2}} \end{aligned}\right\} \qquad (16)$$

and from Fig. 23 we get

$$\tan \theta = \frac{X}{R} \qquad (17)$$

That is to say, the electromotive force required to maintain a given alternating current I (ammeter value) in the circuit shown in Fig. 21 has a voltmeter value E which is given by equation (16) and it leads the current in phase by the phase angle θ which is given by equation (17). Equations (16) and (17) are together exactly equivalent to equation (*d*), and they are very much simpler to use.

The quantity X as given by equation (15) is called the *reactance* of the circuit, and the quantity $\sqrt{R^2 + X^2}$ is called the *impedance* of the circuit. Reactance and impedance are both expressible in ohms.

PROBLEMS.

23. A harmonic alternating current, maximum value 100 amperes, frequency 60 cycles per second, flows through a circuit consisting of a non-inductive resistance of 2 ohms, a resistance-less inductance of 0.003 henry, and a condenser of which the capacity is 0.00006 farad, all connected in series. Draw a clock diagram representing the phases of the electromotive forces across the resistance, across the inductance and across the condenser, respectively, and calculate the effective value of each. Ans. 141.4 volts across the resistance, 79.97 volts across the inductance, and 3,126 volts across the condenser.

24. (*a*) Calculate the reactance value of 0.12 henry at 60 cycles per second. (*b*) Calculate the reactance value of a 20-microfarad condenser at 60 cycles per second. (*c*) Calculate the reactance of both in series. (*d*) At what frequency is the reactance value of both in series equal to zero?

25. A coil of wire having a resistance of 2 ohms is connected across 110-volt, 60-cycle supply mains and the current flowing through the coil as measured by an ammeter is 10.6 amperes. What is the inductance of the coil in henrys? Assume electromotive force and current to be harmonic.

26. A condenser is connected to 110-volt, 60-cycle supply mains and the current flowing as measured by an ammeter is 1.23 amperes. Calculate the capacity of the condenser in

microfarads, assuming resistance and inductance of connecting wires to be negligible. How much would the resistance of the connecting wires have to be to produce a one per cent under-estimation of the capacity of the condenser?

27. A harmonic electromotive force of 1,100 volts (effective) produces a harmonic current of 100 amperes (effective) in a given circuit, the current being 20° behind the electromotive force in phase. Calculate the resistance, reactance, and impedance of the circuit.

Note.—In all of the following problems effective or voltmeter values of electromotive force, and effective or ammeter values of current are to be understood.

28. A harmonic electromotive force of 110 volts produces a current of 15 amperes in a receiving circuit of which the power factor is equal to 0.7. Find the values of resistance, reactance, and impedance.

Note.—From the data of this problem it is impossible to determine whether the current is ahead of or behind the electromotive force in phase. It is desirable always in specifying the value of the power factor of a receiving circuit to state whether the current leads or lags, thus one would speak of a 70 per cent. leading power factor or a 70 per cent. lagging power factor, as the case may be.

---

29. Electric resonance.—When an alternating mechanical force of frequency f is exerted on a pendulum, the pendulum soon settles to a *steady state of oscillation at frequency* f, and this steady state of oscillation will be most violent if the frequency f of the alternating force happens to coincide with the frequency of free oscillation of the pendulum. This phenomenon is called *resonance*, and the circuit of Fig. 21 shows an exactly analogous phenomenon which is called *electric resonance*.

A harmonic alternating electromotive force of *given value* (as measured by a voltmeter) will produce a maximum of alternating current (as measured by an ammeter) in a circuit like Fig. 21 when the freuqency is such as to make

$$X = \omega L - \frac{1}{\omega C} = 0$$

or

$$\omega = \sqrt{\frac{1}{LC}}$$

This expresses the critical frequency in radians per second; and, since $\omega = 2\pi f$, we have

$$f = \frac{1}{2\pi}\sqrt{\frac{1}{LC}} \tag{18}$$

where f is the critical frequency in cycles per second, or the frequency of free oscillation of the circuit in Fig. 21 as given by equation 36, Art. 78 of Chap. V.

At the critical frequency the reactance X is zero, and equations (16) and (17) reduce to

$$\left.\begin{aligned} E &= RI \\ \text{or} \qquad I &= E/R \end{aligned}\right\}$$

and

$$\tan\theta = 0$$

That is to say, at the critical frequency the reactance of the condenser $(-1/\omega C)$ annuls the reactance of the inductance (ωL), and the voltage-current relation *after the full current is established* is the same as if the circuit contained resistance only.

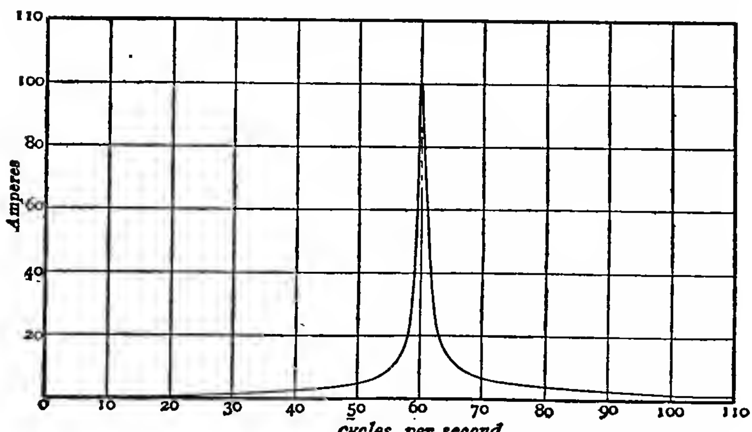

Fig. 24.

The very sharply defined frequency at which resonance occurs is shown in Fig. 24. The ordinates of the curve represent the ammeter values of the current produced by a harmonic electromotive force of 200 volts (voltmeter value) in a circuit like Fig. 21 in which $R = 2$ ohms, $L = 0.352$ henry, and $C = 20$ microfarads, and the abscissas represent various frequencies. At very low frequencies the value of $1/\omega C$ is very great, and the current is limited by the condenser reaction; at the critical frequency of 60 cycles per second, the current is equal to E/R ($= 100$ amperes ammeter value); and at very high frequencies the value of ωL is very great, and the current is limited by the inductance reaction.

Multiplication of electromotive force by resonance.—When an inductance and a condenser are connected in series and to alternating supply mains, the two parts a and b, Fig. 15a, into which the supply voltage is subdivided may each be much larger than the supply voltage itself. This is called the *multiplication of electromotive force by resonance.*

The multiplication of electromotive force by resonance may be easily understood in terms of its mechanical analog as follows: A flat spring is clamped in a vise and a heavy weight is fixed to the free end of the spring. An alternating force acts on the weight and, in general, part of this force is used to accelerate the weight and part of it is used to bend the spring—but when the frequency of the force is the same as the frequency of the free oscillations of the weight *and after the oscillations have become steady*, then the outside force is used only to overcome friction (including what may be called molecular friction in the spring); the acceleration and deceleration of the weight is taken care of by the elasticity of the spring, and the bending of the spring is taken care of by the inertia reaction of the weight.

Multiplication of current by resonance.—When an inductance and a condenser are connected in parallel the total current delivered by the supply mains to the combination may be much smaller than the current in either. The two parts a and b into

which the total current c is divided, see Fig. 16*a*, may each be much larger than the current c. This is called the *multiplication of current by resonance.*

The multiplication of current by resonance may be easily understood with the help of Figs 25 and 26. The clock-diagram

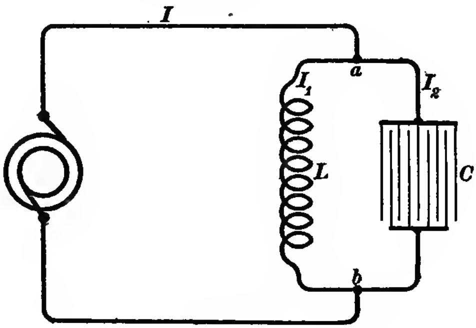

Fig. 25.

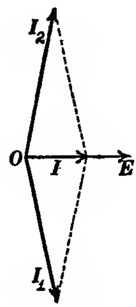

Fig. 26.

vector OE in Fig. 26 represents the voltage across the branch points a and b in Fig. 25, the vector I_1 represents the lagging current flowing through L, the vector I_2 represents the leading current through C, and the vector I represents the total current delivered to the combination.

Although L and C are in *parallel* with each other in Fig.

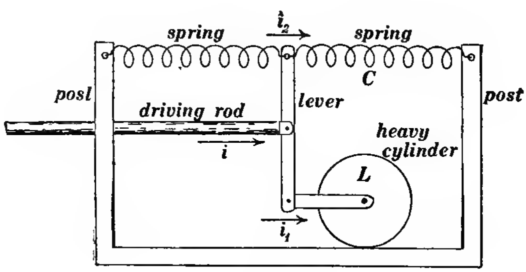

Fig. 27.

25 in their relation to the supply mains, they are in *series* with each other in the closed circuit $LaCb$, and multiplication of

current by resonance is due to the surging of current back and forth in this closed circuit.

The multiplication of current by resonance may be strikingly shown by connecting very small low-voltage glow lamps in the main circuit and in the two branches in Fig. 25. Then if $\omega L = 1/\omega C$ the two lamps in the branch circuits will be heated to full brilliancy while the single lamp through which the undivided current flows will not be perceptibly heated.

The multiplication of alternating current by resonance may be most easily understood in terms of the mechanical analog as follows:

The to and fro velocity i of the driving rod of Fig. 27 divides up into the two velocities i_1 and i_2 of the ends of the lever.	The main current i in Fig. 25 divides up into two currents i_1 and i_2 in the two branches L and C.
The velocity i of the driving rod is at each instant equal to *half* the algebraic sum of the velocities i_1 and i_2 of the ends of the lever.	The current i in the main circuit is at each instant equal to the *whole* algebraic sum of the currents i_1 and i_2 in the two branches.
At a critical frequency the to and fro motion of the driving rod sets up a see-saw motion of the lever, and the to and fro velocities of the ends of the lever i_1 and i_2 may both be much larger than i.	At a critical frequency the main current i sets up a "see-saw" (a surging of current back and forth around the closed loop $LaCb$), and the branch currents i_1 and i_2 may both be much larger than i.

PROBLEMS.

29. An inductance of 0.08 henry and a condenser of 25 microfarads capacity are connected in series, the resistance of the circuit being 7 ohms. (*a*) What is the critical frequency of the circuit, that is to say, at what frequency does resonance occur?

(*b*) What is the effective value of the electromotive force across the condenser terminals when an effective electromotive force of 110 volts at the critical frequency acts upon the circuit?

30. An inductance of 0.176 henry and a capacity of x microfarads are connected in series to 110 volt 60 cycle supply mains. What must be the value of x to give resonance? What resistance must the circuit have in order that voltage across condenser may be 10 times as great as supply voltage when resonance is established? What resistance in order that voltage across condenser may be 100 times supply voltage?

31. An inductance of 0.176 henry and a capacity of x microfarads are connected in parallel to 110-volt 60-cycle supply mains. Find value of x to give resonance. Find resistance of lamps to be placed in each branch (lamps, only, being supposed to have resistance) so as to make current in each branch 25 times total current supplied to the combination after resonance is established.

30. **Circuits in series. Voltage drop in a transmission line.**— A transmission line is of course in series with the circuit to which it delivers current, and a consideration of the problem of voltage drop in a transmission line will furnish an example of the general problem of circuits in series.

(*a*) *Voltage drop in a transmission line which delivers current to a non-inductive receiving circuit.*—Let OI, Fig. 28, represent

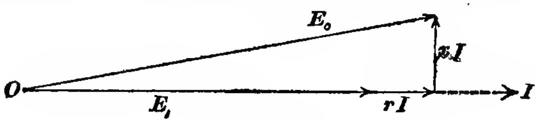

Fig. 28.

the current flowing through the transmission line and the receiving circuit. The voltage E_1 across the terminals of the receiving circuit is in phase with I, and the generator voltage E_0 is the vector sum of E_1, rI and xI; where rI is the electromotive force used to overcome the line resistance r, and xI is the electromotive force used to overcome the line reactance x. An

examination of the figure shows that in this case (receiving circuit non-inductive) the line resistance produces a difference in *value* between E_0 and E_1, whereas the line reactance produces a difference in *phase* between E_0 and E_1.*

(*b*) *Voltage drop in a transmission line which delivers current to a highly inductive receiving circuit of small resistance.*—Let OI, Fig. 29, represent the current flowing through the transmission

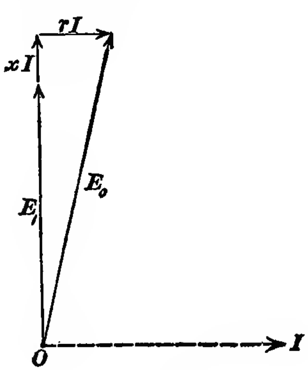

Fig. 29.

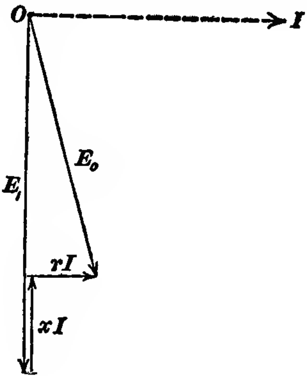

Fig. 30.

line and the receiving circuit. The voltage E_1 across the terminals of the receiving circuit is nearly 90° ahead of I in phase as shown, and the generator voltage E_0 is the vector sum of E_1, rI and xI as before. In the present case, however, the line resistance produces a difference in *phase* between E_0 and E_1, and the line reactance produces a difference in *value* between E_0 and E_1.†

(*c*) *Voltage drop in a transmission line which delivers current to a condenser or any receiving circuit having negative reactance, but having small resistance.*—This case is shown in Fig. 30, and E_0 is the vector sum of E_1, rI and xI as before; but, E_1 and xI are nearly opposite in direction, inasmuch as E_1 is

* This statement is made on the assumption that xI is very small as compared with E_1.

† This statement is made on that assumption that rI is very small as compared with E_1.

nearly 90° behind I under the assumed conditions. Therefore in this case the line resistance produces a difference in *phase* between E_0 and E_1 and the line reactance produces a difference in *value* between E_0 and E_1, **making E_1 greater than E_0.**

(*d*) *Voltage drop in a transmission line which delivers current to a receiving circuit whose power factor (cos θ) has any given value.*—Let the line OI, Fig. 31, represent the current flowing

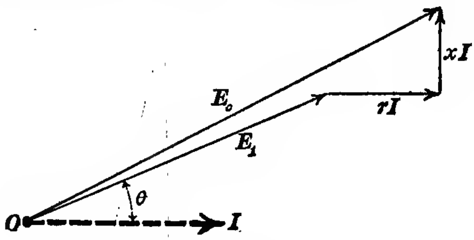

Fig. 31.

through the transmission line and the receiving circuit. Let the line E_1 represent the voltage across the receiving circuit, θ being the phase difference between I and E_1 as shown. Let r be the resistance and x the reactance of the transmission line, and let E_0 be the voltage at the generator. Lay off rI parallel to OI, and xI at right angles to OI. Then the difference between the values of E_1 and E_0 is the required transmission line drop.

The value of $\sqrt{r^2I^2 + x^2I^2}$ is called the *impedance drop*, and the difference between the numerical values of E_0 and E_1 is called simply the (*voltage*) *drop* in the line.

31. **Circuits in parallel.**—The general problem of circuits in parallel is somewhat complicated if one attempts to find algebraic expressions for the combined resistance and reactance of two or more circuits in parallel. Two simple cases of this problem are, however, of considerable interest, namely, (*a*) the problem of finding the power factor of two receiving circuits in parallel, the power factor of each and the current delivered to each being given, and (*b*) the problem of compensating for lagging currents.

(*a*) *Power factor of receiving circuits in parallel.*—Let the line

OE, Fig. 32, represent the voltage across the two receiving circuits. Let I_1 be the current delivered to receiving circuit number one and $\cos\theta_1$ its power factor, and let I_2 be the current delivered to receiving circuit number two and $\cos\theta_2$ its power factor. Lay off the clock diagram in Fig. 32 carefully to scale, and draw the diagonal I as shown. The angle θ between E and I is then the angle whose cosine is the required power factor of the two receiving circuits in parallel.

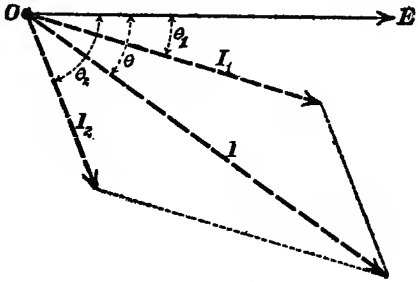

Fig. 32.

(*b*) *Compensation for lagging currents.*—When a transmission line delivers current to an inductive receiving circuit, power is delivered over the line to the receiving circuit during the time that the product ei is positive, and power is delivered back over the line from the receiving circuit to the generator during the time that the product ei is negative (compare Art. 23). This backward flow of energy from receiving circuit to generator represents an essentially unnecessary service of the transmission line, and it is desirable, and in some cases feasible, to reduce this backward flow of energy by connecting a condenser or something which is equivalent to a condenser (such as an over-excited synchronous motor) in parallel with the receiving circuit.

Let OE, Fig. 33, represent the voltage at the terminals of the receiving circuit, and let OI represent the lagging current delivered to the receiving circuit. Let R be the resistance of the receiving circuit and X its reactance. Then

$$\tan\theta = \frac{X}{R}$$

and

$$\sin\theta = \frac{X}{\sqrt{R^2 + X^2}}$$

The component Ob of the current I is equal to $I \sin \theta$ or $IX/\sqrt{R^2+X^2}$. But I is equal to $E/\sqrt{R^2+X^2}$, so that the component Ob is equal to $EX/(R^2+X^2)$. If a condenser of capacity C is connected across the transmission line at the receiver end, then the current flowing into the condenser will be 90° ahead of E, or parallel to Oc, Fig. 33, and its value will be

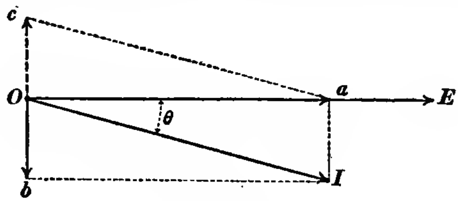

Fig. 33.

equal to E divided by the condenser reactance $1/\omega C$, or to $E\omega C$; and if this current is numerically equal to Ob, the sum of I and Oc will be in phase with E. That is, the receiving circuit and the condenser together will take current in phase with E, and the instantaneous value of power delivered over the transmission line will never be negative. The capacity of the condenser required to produce this result is given by the relation above mentioned, namely, $E\omega C = EX/(R^2+X^2)$, which gives

$$C = \frac{X}{\omega(R^2+X^2)}$$

PROBLEMS.

32. A transmission line of which the resistance is 4 ohms and the reactance is 3 ohms delivers 100 amperes to a non-inductive receiving circuit, the electromotive force across the terminals of the receiving circuit being 10,000 volts. (*a*) What is the value of the generator voltage, and (*b*) what is the phase difference between generator voltage and the voltage across the receiving circuit? Ans. (*a*) 10,404.3 volts, (*b*) 1°39′.

Note.—The answer as given is absurdly precise, the last three digits would be meaningless under practical conditions. This remark applies also to the answers given for problems 33 and 34.

33. If the transmission line specified in problem 32 were to deliver 100 amperes of current at 10,000 volts to a condenser, what would be the value of the generator voltage? Ans. 9,703.1 volts.

34. The transmission line specified in problem 32 delivers 100 amperes of current at 10,000 volts to a receiving circuit of which the power factor (lagging) is 0.7. What is the value of the generator voltage? Ans. 10,494.6 volts.

35. An alternator delivers current to two inductive receiving circuits in parallel. One receiving circuit takes 20 amperes and its power factor (lagging) is 0.9, and the other receiving circuit takes 25 amperes and its power factor (lagging) is 0.7. What is the power factor of the combination?

36. An alternator delivers 100 amperes at 1,100 volts and 60 cycles per second to an inductive receiving circuit of which the power factor is 0.85. What capacity condenser would be required to compensate for lagging current? What number of leaves of paraffined paper 22 by 27 centimeters would be required for this condenser, thickness of paraffined paper being 0.08 centimeter, allowing 1 centimeter margin beyond the tinfoil? Take inductivity of paraffined paper equal to 2. Ans. 127.0 microfarads, 114,800 leaves.

APPENDIX C.

ELECTRICAL MEASUREMENTS.

32. **International standards.**—The international standard ampere, as defined in Art. 24 of Chapter II, is based on a very careful determination by Lord Rayleigh of the deposition of metallic silver from a solution of pure silver nitrate by a current measured in terms of its magnetic effect.

The international standard ohm, the resistance at 0° C. of a column of pure mercury 106.3 centimeters long of uniform sectional area and weighing 14.4521 grams, is based on very careful measurements by the method of Lorentz as outlined in problem 80 on page 85.

33. **Working standards.**—The international standards as above defined are very inconvenient to use in the laboratory.

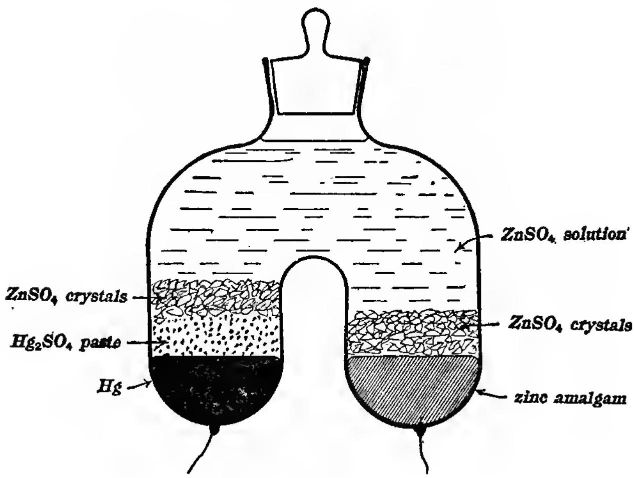

Fig. 34.

Nearly all precise measurements in the laboratory are based on working standards as follows.

Resistance standards are made of carefully annealed wire, usually manganin wire, with heavy copper terminals; and when certified by the United States Bureau of Standards they can be depended upon certainly to one part in ten thousand.

Electromotive force standards.—The Clark standard cell, which is shown in cross-section in Fig. 34 is extensively used as a

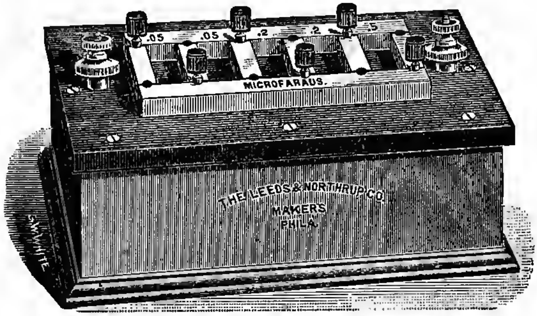

Fig. 35.

standard of electromotive force,* and its electromotive force when it is at a temperature of $t°$ C. is:

$$E \text{ (in volts)} = 1.4292 - 0.00123\,(t - 18) - 0.000{,}007\,(t - 18)^2$$

The Weston standard cell is similar in every respect to the Clark cell except that cadmium amalgam and cadmium sulphate are used instead of zinc amalgam and zinc sulphate. Its electro-

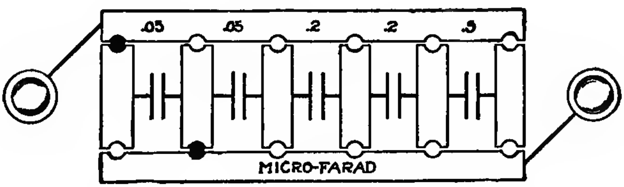

Fig. 36.

motive force (with saturated solution of cadmium sulphate) when it is at a temperature of $t°$ C. is

* Specifications for the Clark cell are given on page 12 of Franklin, Crawford and MacNutt's *Practical Physics*, Vol. II.

$$E \text{ (in volts)} = 1.0187 - 0.000035\,(t - 18) - 0.000{,}000{,}65\,(t - 18)^2$$

Standard condensers.—A general view of a subdivided condenser is given in Fig. 35, and a diagram of the internal connections is shown in Fig. 36. Such condensers when carefully made, using mica as the dielectric, and certified by the Bureau of Standards may be depended on, certainly, to one tenth of one per cent.

Standard inductances.—A fixed standard of inductance is a winding of insulated copper wire wound preferably on a marble

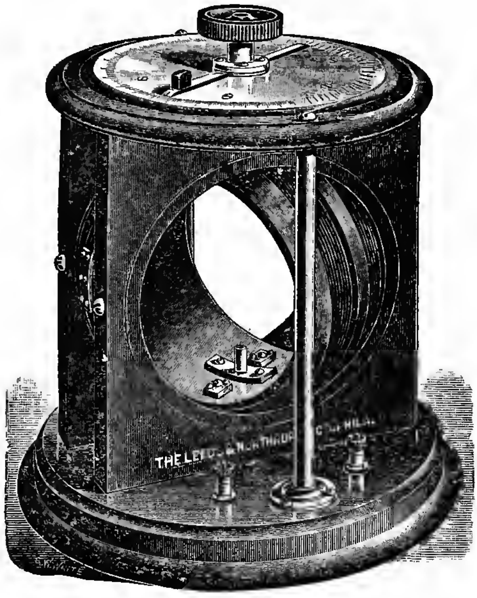

Fig. 37.

spool. A variable standard of inductance consists of a fixed coil of wire with an inside coil that can be turned so that the coils may stand with any desired angle between their planes. The two coils are connected in series and the inductance of the two coils in series is indicated by a pointer which plays over a divided circle. A variable inductance of this type is shown in Fig. 37.

34. The potentiometer.—The potentiometer is an arrangement

for measuring the ratio of two electromotive forces as follows: A battery B produces a constant current i through a stretched wire WW, Fig. 38, and the sliding contacts a, a', A and A'

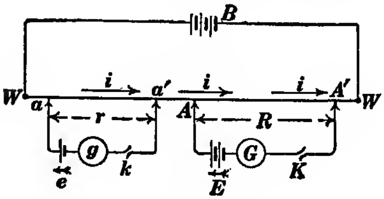

Fig. 38.

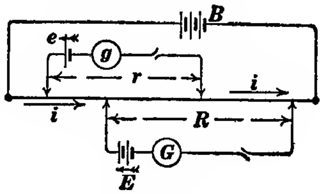

Fig. 39.

are moved until neither galvanometer gives a deflection when keys k and K are closed Then $e = ri$ and $E = Ri$, whence $E/e = R/r = L/l$, where L is the measured length of the portion AA' of the wire WW and l is the measured length of the portion aa'. Therefore, if e is the known electromotive force of a standard cell, the value of E can be calculated.

The above described potentiometer is called the *simple slide wire potentiometer*, and it is not very accurate because the wire WW is sure to be to some extent non-uniform so that R/r is not equal to L/l. This source of error is very greatly reduced in the Leeds and Northrup potentiometer by using a very care-

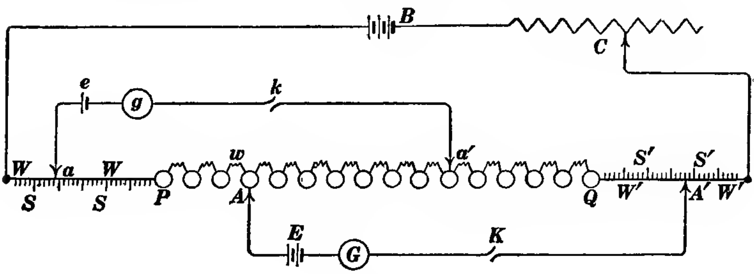

Fig. 40.

fully drawn slide wire and by arranging so that the slide wire is a small portion, only, of the two resistances r and R. The essential features of the Leeds and Northrup potentiometer are

shown in Fig. 40, and it is to be noted: 1st that Fig. 39 is the exact equivalent of Fig. 38, and 2nd that one galvanometer, only, is needed in Fig. 38 or Fig. 39 if a change-over switch (see Fig. 4 of Chapter I) is used to transfer the galvanometer quickly from g to G or from G to g.

Fifteen resistances each equal to w ohms are connected between sixteen metal blocks which are represented by the small circles in Fig. 40; WW is a wire on which is a sliding contact a, and the reading of a on the scale SS gives the resistance aP in thousandths of w; and $W'W'$ is a wire on which is a sliding contact A', and the reading of A' on the scale $S'S'$ gives the resistance QA' in thousandths of w. Thus the values of r and R of Fig. 39 can be read off accurately in terms of w in Fig. 40.

The Leeds and Northrup potentiometer is ordinarily used as a direct reading instrument as follows: Suppose a Weston cell is used for e and suppose that the temperature of the Weston cell is 22° C. Then $e = 1.01855$ volts. Set contact a' so as to include $10w$ between P and a', and set contact a so that $aP = 0.1855w$; and then adjust the rheostat C until galvanometer g gives no deflection. Then adjust contacts A and A' until galvanometer G gives no deflection, and read off resistance R (between A and A'). Suppose, for example, that $R = 12w + 0.2256w$. Then

$$\frac{E}{e} = \frac{12.2256w}{10.1855w}$$

or, since $e = 1.01855$ volts, we have

$$E = \frac{12.2256}{10.1855} \times 1.01855 = 1.22256 \text{ volts.}$$

That is to say, the reading for R divided by 10 gives E in volts.

A carefully adjusted potentiometer carefully used should give a precision of 1/50 of one per cent. so that the last digit in the result 1.22256 is meaningless.

Figure 41 is a general view of a Leeds and Northrup potentiometer. The slide wire $W'W'$ is wound on a cylinder, and the slide wire WW of Fig. 40 is replaced by a step by step resistance. The above description is sufficient to give the reader a clear understanding of the Leeds and Northrup potentiometer, but it is

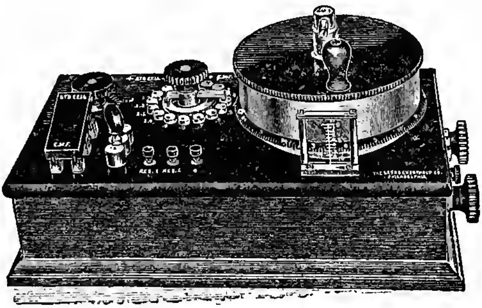

Fig. 41.

advisable to follow the manufacturer's directions in using the instrument. By using an auxiliary resistance box a high voltage may be measured (see manufacturer's directions).

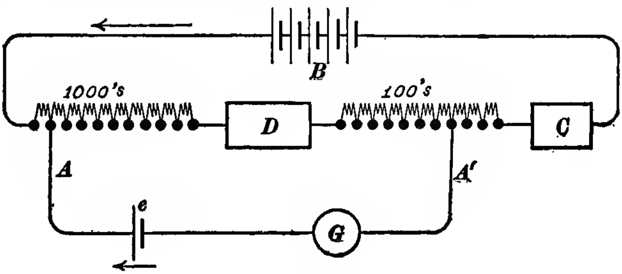

Fig. 42.

Another design of potentiometer is that of Otto Wolff. This instrument is shown diagrammatically in Fig. 42. The branch circuit which contains the standard cell e and the galvanometer g (the electromotive force E may be quickly put in place of e in this branch thus making it the branch EG in Fig. 39) is connected to two movable arms A and A' which play over rows

of contact blocks so as to include any desired number of thousand-ohm coils and any desired number of hundred-ohm coils between A and A'. In order to adjust the resistance between A and A' by ten-ohm steps, by one-ohm steps and by tenth-ohm steps *without altering the total resistance in circuit with battery* B, an arrangement is used to insert ten-ohm coils at D and take out ten-ohm coils at C, or vice versa; another similar arrangement is used to insert one-ohm coils at D and take out one-ohm coils at C, or vice versa; and another similar arrangement is used to

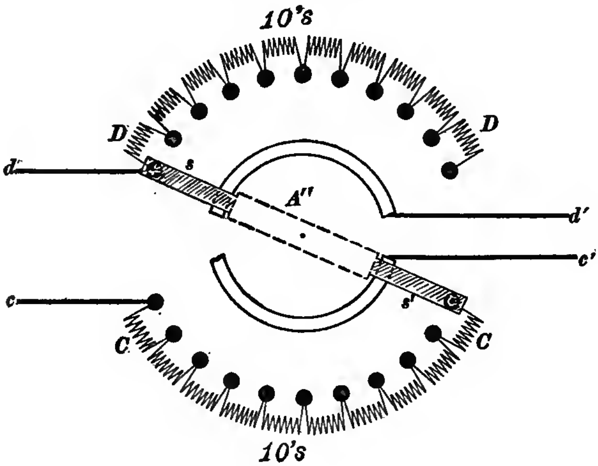

Fig. 43.

insert tenth-ohm coils at D and take out tenth-ohm coils at C, or vice versa. The details of one of these arrangements for inserting coils at D and taking out coils at C, or vice versa, are shown in Fig. 43, in which DD and CC are two sets of ten-ohm coils. The circuit through D of Fig. 42 is from d through s to d' in Fig. 43, and any desired number of ten-ohm coils may be included in this circuit according to the position of the connector s. The circuit through C, Fig. 42, is from c through s' to c' in Fig. 43, and the number of ten-ohm coils which are included in this circuit depends on the position of the connector

s'. The two connectors are carried on a pivoted arm A''. A top view of the Wolff potentiometer is shown in Fig. 44. The auxiliary battery B is connected to binding posts B in Fig. 44, the standard cell e is connected to binding posts N, the electromotive force E is connected to binding posts X, and the gal-

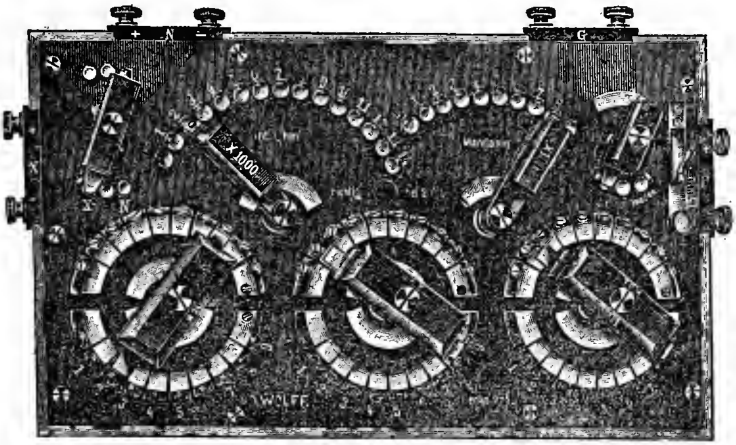

Fig. 44.

vanometer G is connected to posts G. The switch at the upper left-hand corner in Fig. 44 is a change-over switch (see Fig. 4 of chapter I) to connect either e or E in the branch circuit in Fig. 42. The switch at the upper right-hand corner in Fig. 44 is for connecting more or less resistance in the branch circuit in Fig. 42 (the galvanometer circuit) during the earlier stages of each adjustment for zero deflection.*

To standardize a voltmeter connect electromotive force E to voltmeter and to binding posts X in Fig. 44, measure E by means of the potentiometer and observe the corresponding voltmeter reading.

To standardize an ammeter connect a standard resistance R in series with the ammeter and pass a steady current I through

* There are four schemes for using the Wolff potentiometer. See Franklin, Crawford and MacNutt's *Practical Physics*, Vol. II, pages 66–71.

both. Connect terminals of R to binding posts X in Fig. 44 and measure the voltage $E = RI$. Then I is known and the corresponding ammeter reading is observed.

To standardize a wattmeter connect a steady electromotive force E (measured by a standardized voltmeter) across A and R of Fig. 20, Appendix B, send steady current I measured by a standardized ammeter) through coil B of Fig. 20, and take the reading of the wattmeter. The product EI is then the true watts corresponding to this reading.

Standardization of alternating-current ammeters, voltmeters and wattmeters.—The electrodynamometer type of ammeter and voltmeter and the electrodynamometer wattmeter (see Arts. 15 and 24 of Appendix B) are always, or nearly always, used for precise alternating-current measurements, and these instruments indicate correctly* as alternating-current instruments when they have been standardized as direct-current instruments.

35. **Special methods for measuring resistance.**—(*a*) *The ammeter-voltmeter method.*—An ammeter is connected to direct-current supply mains in series with an unknown resistance R and a controlling rheostat. The current I is measured by the ammeter and the voltage E across the terminals of R is measured by a voltmeter. Then $E = RI$ or $R = E/I$. This simple formula must be modified when an accurate result is desired, because (*a*) When the voltmeter is connected across R the ammeter reads combined current through R and through voltmeter, and (*b*) When the voltmeter is connected across ammeter and R the voltmeter reads combined voltage drop in ammeter and in R. Let the student work out the necessarv formulas for himself.

(*b*) *Measurement of insulation resistance.*—Current from the battery B in Fig. 45 flows through the water in the vat V, *through the insulating covering of a coil of rubber covered wire*, through the galvanometer G, through the key K (when it is

* Certain conditions as to inductance being satisfied, as explained in Arts 15 and 24 of Appendix B.

closed), and through the safety resistance R. Then $E = R_t I$ or $R_t = E/I$, where E is the known electromotive force of the battery, I is the current as measured by the galvanometer, and R_t is the total resistance of the entire circuit. Usually the

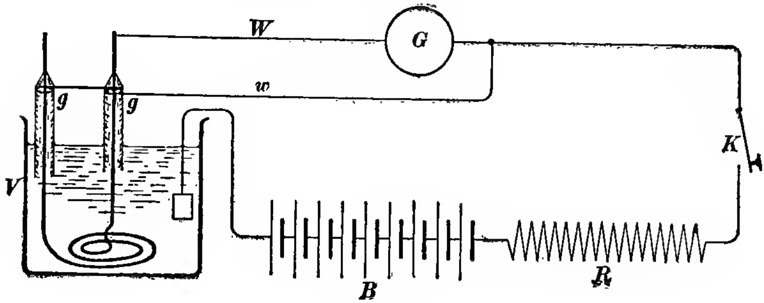

Fig. 45.

resistance of the insulating covering of the wire is so large that all other resistance in the circuit is negligible in comparison so that R_t may be taken to be the resistance of the insulating covering without appreciable error.

Some current leaks over the surface of the insulation, and the guard wire *ggw* is connected to carry this current *around* the galvanometer so that the current which is indicated by the galvanometer may be the current which actually flows through the rubber covering of the wire.

PROBLEMS.

37. A voltmeter which has a resistance of 3,000 ohms and an ammeter which has a resistance of 0.01 ohm are used to measure, an unknown resistance R by the ammeter-voltmeter method and the voltmeter is connected so as to measure combined voltage drop across ammeter and R. The voltmeter reading is 32.1 volts and the ammeter reading is 25.5 amperes. Find the value of R.

38. The voltmeter connections in the previous problem are changed so that the ammeter reads total current delivered to

R and to voltmeter. The voltmeter reading is 32.1 volts what is the ammeter reading?

39. A gravity battery cell of which the electromotive force is 1.07 volts and of which the resistance is 2 ohms sends current through a 2,000-ohm rheostat and through a parallel arrangement of a sensitive galvanometer (800 ohms resistance) and a two-ohm shunt, and the galvanometer deflection is observed to be 21.4 scale divisions. How much current does one division of galvanometer deflection represent? No need to calculate this result closer than 4 or 5 per cent.

The above galvanometer when arranged as indicated in Fig. 45 gives 4.1 divisions deflection, the electromotive force of the battery B being 220 volts. Find the value of R_t (see Art. 35 above). If a precision of 5 per cent. is all that can be realized, how much might the resistance of water, connecting wires and R in Fig. 45, be, and yet be negligible in comparison with the resistance of the rubber covering?

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36. Uses of the ballistic galvonameter.—The two fundamental uses of the ballistic galvanometer, namely, for the measurement of electric charge and for the measurement of magnetic flux, are briefly described and explained in Art. 71 of Chapter V.

Standardization of ballistic galvanometer by means of a standard cell and a standard condenser.—A standard condenser of which the capacity C is accurately known is charged by a known electromotive force (the electromotive force of a standard Clark cell, for example) and discharged through the ballistic galvanometer, and the throw d of the galvanometer is observed. Then $CE = kd$ whence $k = CE/d$.

Standardization of ballistic galvanometer by change of magnetic flux.—The reduction factor $k = CE/d$ as above determined may be used for the measurement of magnetic flux as explained in Art. 71 of chapter V, but it is sometimes more convenient to standardize the ballistic galvanometer as follows: A single layer

of Z' turns of wire is wound on a long wooden cylinder of radius r, and the magnetic flux through the wooden rod is $\Phi = 4\pi \frac{Z'}{l} I \times \pi r^2$, where l is the length in centimeters of the winding of Z' turns, and I is the current in the winding in abamperes as measured by an ammeter. An auxiliary winding of Z turns is wound around the middle of the long coil and connected to the ballistic galvanometer, and the throw d of the ballistic galvanometer is observed when the circuit of the long coil is broken and the current I is reduced to zero. The change of flux is, of course, equal to $4\pi \frac{Z'}{l} I \times \pi r^2$, and therefore, according to equation (iii) of Art. 71 of chapter V, we have

$$4\pi \frac{Z'}{l} I \times \pi r^2 = \frac{Rkd}{Z}$$

from which the value of k may be calculated, R being the known resistance of the ballistic galvanometer circuit in abohms.

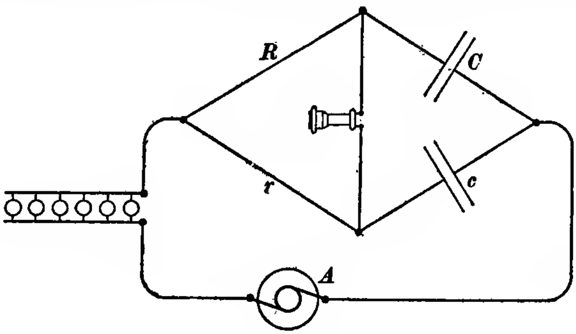

Fig. 46.

37. Measurement of capacity and measurement of inductance by means of Wheatstone's bridge.*—(*a*) *Comparison of capacities.* There are several arrangements for using a Wheatstone's bridge for comparing two capacities, and one of the

* Simple alternating-current methods for measuring capacity and inductance are very briefly outlined in problems 25 and 26 of Appendix B.

simplest arragements is shown in Fig. 46. The two condensers C and c to be compared are connected as shown, alternating current is supplied by the alternator A, and the ratio arms R and r of the bridge are non-inductive. The ratio R/r is adjusted until the telephone gives a minimum sound, and then $C/c = R/r$.

(*b*) *Comparison of inductances.*—The inductance X to be measured is connected in series with an adjustable non-inductive resistance r in one arm of a Wheatstone bridge as shown in Fig. 47, a variable standard of inductance S is connected in the

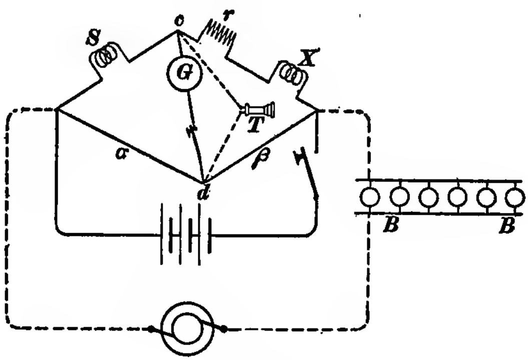

Fig. 47.

other arm of the bridge as shown in Fig. 47, and two adjustments are made as follows: (*a*) The resistances are adjusted until the galvanometer G gives no deflection when a steady battery current flows though the bridge arrangement, and then (*b*) The variable standard of inductance is adjusted until the telephone gives minimum sound when the bridge arrangement is supplied with alternating current as indicated by the dotted lines in the figure. Then $S/X = \alpha/\beta$.

38. Measurement of the horizontal component H of the earth's magnetic field.—(*a*) *Comparison of values of H at two places by magnetometer deflections.* A very small magnet ns, Fig. 48, a pivoted compass needle, is placed where the horizontal

component of the earth's field is H and the needle comes to rest pointing in the direction of H (the plane of the paper is horizontal in Fig. 48). A large bar magnet MM is then placed at a

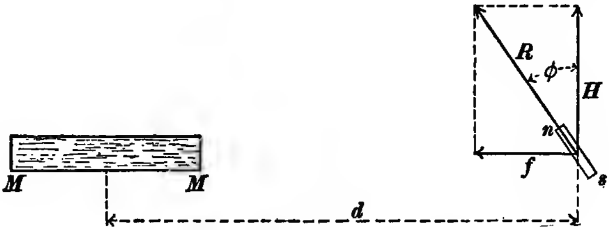

Fig. 48.

measured distance d due magnetic east or west of the compass needle. The large magnet produces at the compass needle a definite westerly (or easterly) field f, and the compass needle is deflected through the angle ϕ, where

$$\tan \phi = \frac{f}{H} \tag{i}$$

The compass needle is then placed where the horizontal component of the earth's magnetic field is H' and the deflection ϕ' produced by the large magnet at the same distance d is observed. Then

$$\tan \phi' = \frac{f}{H'} \tag{ii}$$

and dividing equation (ii) by equation (i), member by member, we get

$$\frac{H'}{H} = \frac{\tan \phi}{\tan \phi'} \tag{iii}$$

(*b*) *Comparison of values of* H *at two places by the method of oscillations.*—The large bar magnet MM of Fig. 48 is suspended and set oscillating at a place where the horizontal component of the earth's magnetic field is H, and we have

$$4\pi^2 n^2 K = mlH \tag{iv}$$

where n is the number of complete vibrations per second, K is the moment of inertia of the suspended magnet referred to the axis of suspension, m is the strength of the poles of the magnet, and l is the length of the magnet (distance between its poles)

The large bar magnet is then suspended at another place where the horizontal component of the earth's magnetic field is H', and we have

$$4\pi^2 n^2 K = mlH' \tag{v}$$

and from equations (iv) and (v) we get

$$\frac{H}{H'} = \frac{n^2}{n'^2} \tag{vi}$$

(*c*) *Gauss's method for measuring* m *and* H.—The value of f in equation (i) is

$$f = \frac{m}{\left(d - \frac{l}{2}\right)^2} - \frac{m}{\left(d + \frac{l}{2}\right)^2} \tag{vii}$$

according to Art. 11 of Chapter I, so that equation (i) becomes

$$\tan \phi = \frac{1}{H}\left[\frac{m}{\left(d - \frac{l}{2}\right)^2} - \frac{m}{\left(d + \frac{l}{2}\right)^2}\right] \tag{viii}$$

and this equation and equation (iv) contain only m and H as unknown quantities so that the values of m and H can be calculated if all other quantities in (viii) and (iv) have been determined. The distance l between the poles of the large magnet is, of course, somewhat indefinite, and this uncertain quantity can be eliminated if the deflection Ψ of the compass needle is observed with the large magnet at distance D. We thus have a third simultaneous equation [in addition to equations (iv) and (viii)], namely,

$$\tan \Psi = \frac{1}{H}\left[\frac{m}{\left(D - \frac{l}{2}\right)^2} - \frac{m}{\left(D + \frac{l}{2}\right)^2}\right] \tag{ix}$$

Writing M for ml, and using equations (iv), (viii) and (ix), we get the two following moderately convenient simultaneous equations for use in determining $M(= ml)$ and H, according to Gauss.

$$\frac{M}{H} = \frac{d^5 \tan \phi - D^5 \tan \Psi}{2\,(d^2 - D^2)}$$

and

$$MH = 4\pi^2 n^2 K$$

39. A simple measurement of radioactivity.—An ideal arrangement for measuring the radioactivity of a weighed quantity of radioactive material is shown in Fig. 49 in which AA and BB are metal plates, MM is a weighed quantity of radioactive material, b is a high-voltage battery and G is a galvanometer for measuring the current. The only difficulty with this arrangement is that the current is usually too small, by far, to be measured by a galvanometer, and therefore the arrangement shown in Fig. 50 must be used. The underlying idea may, however, be most easily set forth by thinking of the arrangement in Fig. 49 as follows: When the voltage of the battery b is large enough* (four or five hundred volts) *all of the electrons and ions which are formed by the radiations from MM serve as current carriers, and the value of the current may be taken as a measure of the radioactivity of MM.*

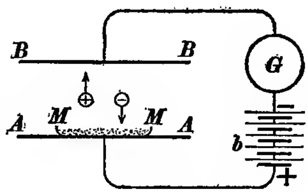

Fig. 49.

Figure 50 is a gold leaf electroscope arranged for the measurement of radioactivity. The metal plate BB, the metal rod R and the gold leaf L are highly insulated, the plug S being of cast sulphur. The plate AA, on which the weighed quantity of radioactive material MM is placed, is connected to ground, the plate BB the rod R and the gold leaf L are charged by

* But not sufficiently large to produce ionization by collision in the region between AA and BB.

momentarily connecting the movable wire C, and the time t required for the gold leaf to fall from a given initial position to a given final position (because of the discharge of plate BB) is

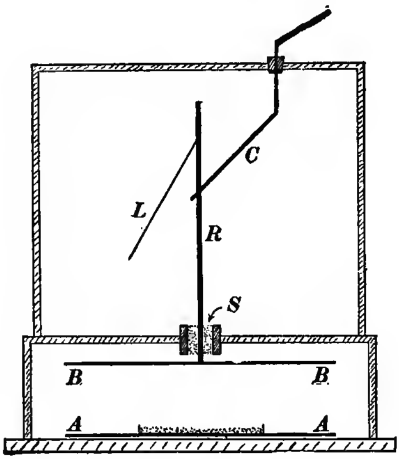

Fig. 50.

observed. The reciprocal of this time t is proportional to the current and it is taken as a measure of the radioactivity of MM.

To correct for incomplete insulation of BB, R and L the time t' required for the given movement of the gold leaf before the material MM is in place is observed, and then $\left(\frac{1}{t} - \frac{1}{t'}\right)$ is the true measure of the radioactivity of MM.

If the object is to study the decay of radioactivity of MM, the value of $\left(\frac{1}{t} - \frac{1}{t'}\right)$ is determined at intervals for several hours or days.

APPENDIX D.

CORRESPONDING EQUATIONS OF TRANSLATORY MOTION, ROTATORY MOTION AND "ELECTRICAL MOTION."

$$x = vt \qquad (1)$$

where x is the distance traveled in t seconds by a body which has a *constant* velocity v

$$\phi = st \qquad (2)$$

where ϕ is the angle in radians turned in t seconds by a body which has a *constant* spin velocity of s radians per second.

$$q = it \qquad (3)$$

where q is the amount of electric charge which flows in t seconds through a circuit in which a *constant* current i is flowing.

$$x = \tfrac{1}{2}at^2 \qquad (4)$$

where x is the distance traveled in t seconds by a body which starts from rest and has a *constant* acceleration a

$$\phi = \tfrac{1}{2}\alpha t^2 \qquad (5)$$

where ϕ is the angle turned in t seconds by a body which starts from rest and has a *constant* spin acceleration α

$$q = \tfrac{1}{2} \times \text{rate of growth} \times t^2 \qquad (6)$$

If the current in a circuit grows at a constant rate starting at zero, the amount of charge which flows through the circuit in t seconds is equal to $\frac{1}{2} \times$ rate of growth of current $\times t^2$.

$$W = Fx \qquad (7)$$

where W is the work done by a force F when the body on which F acts moves distance x in the direction of F.

$$W = T\phi \qquad (8)$$

where W is the work done by torque T when the body on which T acts turns through ϕ radians about the axis of T.

$$W = Eq \qquad (9)$$

where W is the work done by an electromotive force E when an electric charge q flows through the circuit on which E acts.

$$P = Fv \qquad (10)$$

where P is the power developed by a force F when the body on which F acts moves at velocity v in the direction of F.

$$P = Ts \qquad (11)$$

where P is the power developed by a torque T when the body on which T acts turns about the axis of T at a speed of s radians per second.

$$P = Ei \qquad (12)$$

where P is the power developed by an electromotive force E when current i flows through the circuit on which E acts.

$$F = m\frac{dv}{dt} \qquad (13)$$

where $\frac{dv}{dt}$ is the acceleration produced by an unbalanced force F which acts on a body of mass m.

$$T = K\frac{ds}{dt} \qquad (14)$$

where $\frac{ds}{dt}$ is the spin acceleration produced by a torque T which acts on a wheel of which the spin-inertia is K.

$$E = L\frac{di}{dt} \qquad (15)$$

where $\frac{di}{dt}$ is the rate of growth of current due to electromotive force E acting on a circuit of inductance L.

$$W = \tfrac{1}{2}mv^2 \qquad (16)$$

where W is the kinetic energy of a body of mass m moving at velocity v.

$$W = \tfrac{1}{2}Ks^2 \qquad (17)$$

where W is the kinetic energy of a wheel rotating at a speed of s radians per second, the spin-inertia of the wheel about its axis of spin being K.

$$W = \tfrac{1}{2}Li^2 \qquad (18)$$

where W is the kinetic energy of a current i in a circuit of which the inductance is L.

$$F = ax \qquad (19)$$

A ball is fixed to a spring and F is the force required to pull the ball to a distance x from its equilibrium position. The factor a is called the *stiffness coefficient* of the spring.

$$T = b\phi \qquad (20)$$

A body is hung by a wire and T is the torque required to turn the body (and twist the wire) through the angle of ϕ radians. The factor b is called the *coefficient of torsional stiffness* of the wire.

$$q = CE$$

or

$$E = \frac{1}{C} \cdot q \qquad (21)$$

Two metal plates are separated by a layer of dielectric constituting what is called a *condenser*, and E is the electromotive force required to draw q coulombs out of one plate and force it into the other plate. The factor C is called the *capacity* of the condenser.

$$4\pi^2n^2m = a \qquad (22)$$

A ball is fixed to a spring of which the stiffness coefficient is a. Then when the ball is pulled to one side and released it performs simple harmonic motion of which the number of complete vibrations per second is n, m being the mass of the ball.

$$4\pi^2n^2K = b \qquad (23)$$

A body is hung by a wire of which the coefficient of torsional stiffness is b. Then when the body is turned so as to twist the wire and released it performs harmonic rotatory motion of which the number of complete vibrations per second is n, K being the spin inertia of the body referred to the wire as an axis.

$$4\pi^2n^2L = \frac{1}{C} \qquad (24)$$

A charged condenser of capacity C is connected to a circuit of inductance L and of negligible resistance. The discharge from the condenser then surges back and forth through the circuit, we have what is called an oscillatory discharge (harmonic), and the number of complete oscillations per second is n.

$$W = \tfrac{1}{2}\frac{1}{a}F^2 \qquad (25)$$

A spring of which the stiffness coefficient is a is bent, the force which is acting on the fully bent spring is F, and the potential energy of the bent spring is W.

$$W = \tfrac{1}{2}\frac{1}{e}T^2 \qquad (26)$$

A wire of which the coefficient of torsional stiffness is b is twisted, the torque which is acting on the fully twisted wire is T, and the potential energy of the twisted wire is W.

$$W = \tfrac{1}{2}CE^2 \qquad (27)$$

A condenser of which the capacity is C is charged, the electromotive force which is acting on the fully charged condenser is E, and the potential energy of the charged condenser is W.

APPENDIX E.

ANSWERS TO PROBLEMS AND LEADING QUESTIONS.

On page 10:

Problem 8, Ans. 11,550 dyne-centimeters of torque.

1. What is meant by the poles of a magnet?

2. Define the unit pole? How large is the pole or poles supposed to be in this definition? What is meant by a concentrated pole? Ask your professor of mathematics to define a concentrated pole as a finite differential in accordance with his hobby relating thereto.

3. A pole has *m* units of strength. What does this mean?

On page 13:

Problem 9, Ans. 0.0438 centimeter.

Additional problems:

9b. The poles of a bar magnet are 30 centimeter apart and their strength is + 700 units and − 700 units. Find the torque exerted on the magnet when it is placed in a uniform magnetic field of which the intensity is 0.6 gauss, the axis of the bar magnet being inclined 30° to the direction of the field. Ans. 6,300 dyne-centimeters.

9c. A compass balances on its pivot before it is magnetized. Does it balance after it is magnetized? If not, why not?

Problem 10, Ans. 0.0737 per second.

4. What is meant when it is stated that a given region is a magnetic field?

5. The intensity of a magnetic field at a given point is 10 gausses. What does this mean?

6. What is a uniform magnetic field? Give an example. What is a non-uniform magnetic field? Give an example. What is the character of the force action exerted on a complete magnet in a uniform field? In a non-uniform field?

On page 15:

Problem 11, Ans. 2.48 gausses.

" 12, Ans. 13.3°.

" 13, Ans. $H = 0.366$ gauss; $m = 1372$ units pole.

On pages 19–20:

Problem 15, Ans. 7,070,000 dynes.

7. Define the unit of magnetic flux. Why is this unit often called the "line of force"? What is meant by an actual line of force in a magnetic field?

8. The distance d in Fig. 24 is assumed to be very small in comparison with the length and breadth of the polar area (the shaded area) in Fig. 23 side view. Why?

9. Work is done in pulling the magnet poles in Fig. 22 or Fig. 24 apart. Where does this work go? Derive an expression for the energy per unit volume of a uniform magnetic field of intensity H.

On pages 32–34:

Problem 17, Ans. 300 dynes act on S-pole towards the left.
600 dynes act on wire towards the left.

" 20, Ans. 0.6 centimeter.

" 21, Ans. 2,500 dynes attraction.

10. State the right-handed-screw rule concerning the direction of flow of an electric current in a wire. State the right-handed-screw rule concerning the direction of magnetization of an iron rod by a current flowing around the rod.

11. What is the physical explanation of the side push exerted by a magnetic field on a wire in which current is flowing?

12. How would you hold a magnet so as to blow out an electric arc?

13. Why is the current in one of the armature wires on a two-pole direct-current dynamo equal to one half of the current which enters or leaves the armature winding?

14. The current in a wire is 12 abamperes. What does this mean magnetically?

15. What is the legal definition of the ampere? See page 36.

16. A long straight wire carries a current of 25 amperes. What

is the intensity of the magnetic field due to this wire at points distant 12 centimeters from the wire?

17. If m units of north pole were spread uniformly along a length of l centimeters of a very small steel rod, what would be the trend of the lines of force of the magnetic field due to this pole in the region very near the pole? What would be the intensity of this field at a small distance r from the long slender pole? Explain.

18. If the very long slender pole mentioned in question 17 were placed along the axis of a coil or winding of wire on a long paper tube of radius r, what force would be exerted on the pole by the coil? Express this result in terms of the current in the coil in abamperes, the number of turns of wire on unit length of the paper tube and the strength m of the pole, and explain.

19. A wire is at right angles to a magnetic field. State how you can determine the direction of the side push on the wire when the direction of the current and the direction of the field are given.

On pages 42–44:

Problem 24, Ans. 96,540 seconds.
" 25, Ans. 1.07 joules per second per ampere (volts).
" 26, Ans. 3.17 centigrade degrees.
" 27, Ans. 20.3% due to voltaic action.

20. Define the terms electrolysis, electrolyte, electrode, anode and cathode.

21. What kind of an electrolytic cell is a voltaic cell?

22. What is meant by voltaic action and local action in a voltaic cell?

23. What is meant by primary and secondary reactions in an electrolytic cell? Give examples.

On pages 48–49:

Problem 28, Ans. 0.025 ampere per square centimeter.
" 29, Ans. 13.6 hours.
" 30, Ans. 1.51 joules per second per ampere (volts).
" 32, Ans. positive plate gains 95.5 grams; negative gains 143.2 grams.

24. What conditions must be realized in a voltaic cell in order

that it may be used as a storage cell? What about non-disintegration of the electrodes?

25. In what terms is the electrochemical equivalent of silver expressed? What is its value?

26. State the two laws of electrolysis. See I and II on page 46.

On pages 53–54:

Problem 33, Ans. 4.31 ohms; 109.8 joules per second per ampere (volts).
" 34, Ans. 0.00558 centigrade degree per second.
" 35, Ans. 11.25 ohms.
" 36, Ans. 0.2 ohm.
" 37, Ans. 0.122 ohm.
" 38, Ans. 0.00653 ohm.
" 39, Ans. 0.00206 ohm.
" 40, Ans. 0.120 ohm.
" 41, Ans. 8 ohms.

27. State Joule's law.

28. What is an ohm of resistance? What is an abohm?

29. In what terms is resistivity expressed (*a*) When resistance is expressed in ohms, length in centimeters and sectional area in square centimeters, and (*b*) When resistance is expressed in ohms length in feet and sectional area in circular mils? (*c*) What is meant by the conductivity of a substance?

On pages 55–56:

Problem 43, Ans. 4.63 ohms at 0° C.; 6.30 ohms at 90° C.
" 44, Ans. 0.0036 per centigrade degree.
" 45, Ans. 77.3° C.
" 46, Ans. 0.000206 per centigrade degree.

30. In what terms is the temperature coefficient of resistance expressed?

On pages 62–64:

Problem 49, Ans. 85%.
" 51, Ans. 76.2 amperes.
" 52, Ans. 2.75 cents.
" 53, Ans. \$2.63.
" 54, Ans. 115.6 volts.

Problem 55, Ans. 6.2 ohms.

" 56, Ans. 23.5 cents; 4.08 cents.

" 57, Ans. 2.08 cents by gas; 10.7 cents by electric heater.

31. What is meant by the electromotive force E of a battery?

32. Under what conditions is Ohm's law true?

33. Show that the electromotive force of a battery cell is equal to jz, where j is the number of joules of energy developed by the dissolving of one gram of zinc in the cell and z is the number of grams of zinc consumed per second per ampere. On what assumption is this proof based?

34. What is meant when it is stated that a dry cell is polarized?

35. State Joule's law as applied to a portion of a circuit. State Ohm's law as applied to a portion of a circuit. How would you apply equation (12) on page 57 to a portion of a circuit?

36. The international standard ampere is defined on page 36 and the international standard ohm is defined on page 220. Define the volt in terms of these international standards.

37. What conditions must be met in order that one may use the equation $P = Fv$ to calculate the power P developed by a force F? In the first place the velocity v must be parallel to F, but this condition does not interest us here. *The force must act on the body which is moving at velocity* v, *not on some other body, and the force must act while the body is moving.* A horse cannot do work by pulling on a post while a plow is moving, nor can a horse do work by pulling on a plow today if the plow moves only tomorrow! Furthermore, *one cannot, in general, multiply any kind of an average value of* F *during a given time by any kind of an average value of* v *during the same time to get the average power* because the force may have acted chiefly during the first part of the time and the body may have moved chiefly during the last part of the time. Explain why equation (12) on page 57 is not, in general, applicable to a portion of a circuit taking power from alternating-current supply mains.

On pages 65–66:

Problem 58, Ans. 0.151 ampere; 0.753 volt; 0.317 volt.

Problem 59, Ans. 0.543 ohm.
" 60, Ans. 4 ohms.
" 61, Ans. 851 mils.
" 62, Ans. 470 mils.
" 63, Ans. 6,730 volts.
" 64, Ans. 100 mils.

On pages 74–75:

Problem 65, Ans. 24 cells.
" 66, Ans. 15,385 ohms.
" 67, Ans. 50 volts; 40 volts; 20 volts.
" 68, Ans. 0.63 ampere.
" 69*a*, Ans. 0.5 ampere; 0.67 ampere.
" 69*b*, Ans. 0.0606 ampere; 20 ohms.
" 70, Ans. 1/180 ohm.
" 71, Ans. 0.004 ohm.
" 72, Ans. 393,000 ohms.
" 73, Ans. 1.2×10^6 ohms; 48×10^6 ohms.

On pages 83–85:

Problem 74, Ans. (*a*) 8,910 watts; (*b*) 420 watts; (*c*) 104.8 volts; (*d*) 8,490 watts.
" 75, Ans. (*a*) 220 watts; (*b*) 132 watts; (*d*) 66 volts.
" 76, Ans. 660,000 lines or maxwells per second.
" 77, Ans. 942 volts.
" 78, Ans. 1,310,000 lines or maxwells; 851 revolutions per minute.

38. Describe an experiment which shows that an induced electromotive force opposes the flow of current through the armature of an electric motor in operation.

39. What is Lenz's law? Give it in the form first given by Helmholtz.

40. Derive the equation $E = lHv$, and state the physical meaning of each step. (See page 78).

41. Explain the meaning of equation (16) on page 80.

On page 89:

Problem 83, Ans. 11×10^7 lines or maxwells per second; 1100 volts.

Problem 84, Ans. 0.5 ampere; 5 amperes; 1077.5 volts.

" 85, Ans. 0.491 amperes; 1058 volts.

42. Give a diagram showing the essentials of the scheme of winding of a 4-pole alternator armature having 8 armature conductors. Make the diagram similar to Fig. 69.

43. What is meant by the frequency of an alternating current? What is meant by a cycle?

44. Explain step-up and step-down transformation.

45. Two round steel rods have an air gap between their ends as indicated in Fig. 22. A large sheet of copper is placed in this air gap and then the steel rods are suddenly magnetized. Make a sketch of the sheet of copper and draw lines showing the flow of the induced currents in the sheet.

The steel rods being magnetized the large sheet of copper is drawn out of the gap. Make a sketch of the sheet of copper and draw lines showing the approximate flow of the induced currents in the sheet. What force is exerted on these currents by the magnetic field in the gap space and what is the direction of the force? If the velocity of withdrawal of the plate is doubled this force will be doubled. Why?

46. Explain the meaning of Fig. 80.

On pages 98–99:

Note. The electromotive force required to make the current in a coil increase is $e = Z\dfrac{d\Phi}{dt}$, where Z is the number of turns of wire in the coil and $\dfrac{d\Phi}{dt}$ is the rate of increase of magnetic flux through the coil (through the opening of the coil) due to the increasing current. This is evidently true because $Z \times \dfrac{d\Phi}{dt}$ is the back electromotive force induced in the coil according to equation (16) on page 80. *It is only when the growing flux* Φ *is proportional to the increasing current* i *that the conception of* INDUCTANCE *is legitimate.*

Problem 86, Ans. 183 amperes per second; 0.03 second.

" 88, Ans. 83.3 amperes per second.

Problem 89, Ans. 5.26×10^{-5} henrys; 7.6×10^{8} amperes per second.

" 90, Ans. 800 amperes per second.

" 91, Ans. 2.677 amperes.

47. Differentiate equation (22) on page 96, and, using equation (12) on page 57, prove equation (20) on page 95. Make the physical meaning of each step perfectly clear.

48. A circuit carries a current which rises to a maximum of 5 amperes and the resistance of the circuit is 4 ohms. The value of $\frac{di}{dt}$ ranges about 2000 amperes per second and the inductance of the circuit is one henry. Would you consider this circuit to be non-inductive? A circuit may be considered as non-inductive under one set of conditions but by no means as non-inductive under another set of conditions. Explain.

49. Does a coil with an iron core have a definite inductance?

50. What is an induction coil? What is an inductance? What is a choke coil?

51. What becomes of the energy $\frac{1}{2}Li^2$ when an inductive circuit is broken?

On pages 108–109:

Problem 95, Ans. 0.744 ampere.

" 98, Ans. 6.93×10^{-8} abcoulombs per division; $H = 10{,}032$ gausses.

52. What is a condenser? What is meant by charging a condenser? What is meant by discharging a condenser?

53. How does a condenser connected across a break in a circuit eliminate the spark at break? How does the condenser in Fig. 74 cause a quick demagnetization of the iron core of the induction coil?

54. What is a coulomb? What is an ampere-second? What is the difference between an ampere-second and an ampere per second?

55. Describe an experiment showing that two insulated metal plates connected to electric supply mains attract each other.

56. How much electric charge will a one-farad condenser hold?

Is the word *capacity* as applied to a condenser used in a sense which is strictly analogous to its use when we speak of the capacity of a water pail? Explain.

57. Define the farad; the microfarad.

On pages 111–112:

Problem 100, Ans. 654 leaves of mica; 655 leaves of tin foil.
" 101, Ans. 0.015 microfarad.

58. What is meant by the inductivity of a dielectric?

On pages 114–115:

Problem 104, Ans. 8.84×10^{-6} joules; 8.84×10^{-2} joules.
" 105, Ans. (*a*) 20,000 volts; (*b*) and (*c*) 8.84×10^{-2} joules or 8.84×10^{5} ergs; (*d*) and (*e*) 4.42×10^{5} dynes.
" 106, Ans. (*a*) 200 volts; (*b*) and (*c*) 8.84×10^{-6} joules or 88.4 ergs; (*d*) and (*e*) 44.2 dynes.

59. If you charge a condenser by allowing the full charging voltage E to act on it from the start is all the work done by E represented by the potential energy of the condenser? If not, why not? If not, what becomes of the remainder? How would you charge a condenser so that all the work done by the charging electromotive force is represented by the potential energy of the condenser?

60. Prove equation (35) page 113 as applied to a charged condenser and state the physical meaning of each step.

61. What can you say as to the charges $+q$ and $-q$ on two parallel plates if the plates are perfectly insulated? Suppose that two such charged plates are pulled farther apart, prove that the voltage between the plates must increase in proportion to the distance between the plates.

Does the potential energy of the charged plates increase as they are pulled apart? If so where does this increase of energy come from?

Derive an expression for the force of attraction of two parallel metal plates in terms of size and distance of plates, inductivity k of the dielectric and voltage E between them.

How much force is one joule per centimeter?

On pages 120–121:

Problem 109*b*. Ans. 167 joules.

" 109*d*, Ans. 65,000 per second.

62. Under what conditions is the voltage required to puncture a dielectric proportional to the thickness of the dielectric? What do you mean by the specific strength of a dielectric? In what terms is it expressed?

63. If you could never produce a movement more rapid than one foot per century, how could you set a tuning fork vibrating? Think of this question in connection with Art. 77.

On pages 125–126 :

Problem 110. Ans. 1.78×10^{-9} colombs.

64. What is meant by an electric field? What is meant by the intensity f of an electric field at a point? Answer this question in terms of force exerted by the field on a small charged ball. What is meant by the direction of an electric field at a point? In what terms is the intensity of an electric field expressed? What is meant by a line of force drawn through an electric field?

65. The oppositely charged plates in Fig. 95 attract; would you say that electric field has a certain tension like a magnetic field? See pages 18 and 19.

66. The oppositely charged plates in Fig. 95 represent a store of potential energy [see equations (33), (34) and (35) on page 113]. Where does this energy reside? Does a magnetic field represent a store of energy? Where does the kinetic energy of the current, $\frac{1}{2}Li^2$, reside in Fig. 81?

On page 138:

Problem 111, Ans. 1.47×10^{-3} coulomb.

" 112, Ans. 2.6×10^{-5} coulomb.

" 113, Ans. 11,538 volts per cm. in the oil; 3,846 volts per cm. in the glass.

" 114, Ans. (*a*) 330,000 volts; (*b*) 139,500 volts.

67. What is meant, mathematically, by electric flux? By electric flux density? See Art. 90.

68. Prove that the electric field intensity between the plates in Fig. 95 is q/aB volts per centimeter where q is the charge in

coulombs on one of tne plates, a is the area of one face of one plate in square centimeters and B has the value given in Art. 73 or in Art. 90.

69. Knowing the energy of the two charged plates in Fig. 95 [see equations (31) and (33)–(35) on pages 110 and 113], derive an expression for the energy per cubic centimeter of an electric field (in air, let us say) of f volts per centimeter. Show that this energy per cubic centimeter is equal to the tension of the field in units of force per square centimeter.

70. The lines of force of an electric field meet the surface of an insulator at right angles.* The surface has no charge on it, let us suppose. What is the relation between electric flux density just outside and just inside of the insulator? What is the relation between the field intensity (volts per centimeter) just outside and just inside of the insulator?

71. Make a sketch showing the approximate trend of the lines of force near an uncharged glass sphere in an electric field, which, but for the presence of the sphere, would be a uniform field. Ditto for an air bubble in oil.

72. Given two short, square-ended cylinders of rubber. (*a*) The cylinders are set side by side between two parallel jaws and squeezed. Is stress or strain the same in the two cylinders? (*b*) The cylinders are placed end on end between two jaws and squeezed. Is stress or strain the same in the two cylinders? (*c*) Are these mechanical cases "directly" or "inversely" analogous to blocks of insulating material side by side between charged plates or end on end between charged plates?

On pages 140–141:

Problem 116, Ans. (*a*) 708 microfarads; (*b*) 708 coulombs; (*c*) 0.00157 volts per centimeter. *Note.* The radius of the earth is about 6375 kilometers.

" 118, Ans. 5,000 volts per centimeter.

On page 175:

Problem 1, Ans. 22 gausses.

" 2, Ans. 7.5×10^{-4} henry.

* Not necessarily true, merely assumed for the sake of the following questions.

Problem 3, Ans. 2,655 ampere-turns.
" 4, Ans. 37.7 gausses.
" 5, Ans. 4.0 amperes.
" 6, Ans. 1,920,000 maxwells or lines.

73. If a short thick rod of iron is placed in a magnetic field, which, but for the presence of the rod would be a uniform magnetic field of intensity H parallel to the rod, the "magnetizing force" is less than H; but if the iron rod is very long and slender the "magnetizing force" would be equal to H. Explain.

74. Consider a long slender iron rod of sectional area q which is magnetized by being placed in and parallel to a uniform magnetic field of intensity H. Two distinct causes produce magnetic field near one of the poles of the rod, namely, the original cause of the uniform field H and the magnet pole on the end of the rod (of strength m). Show that the total magnetic flux which emanates from the north pole of the rod is $4\pi m + Hq$. This is, of course, the total flux Φ coming along or through the rod to the north pole, so that $\Phi/q = B = 4\pi \frac{m}{q} + H$. The ratio m/q is called the intensity of magnetization of the rod.

On page 184:

Problem 8, Ans. (*a*) 3.44 joules; (*b*) 3.42 joules.
" 9, Ans. 6.63 watts.

75. How is work done in magnetizing a bar of iron by means of a coil of wire?

76. What is meant by magnetic hysteresis?

On page 190:

Problem 10, Ans. (*a*) 22 volts; (*b*) 49.2 volts.

On pages 194–195:

Problem 11, Ans. 0.0694 cycle; 0.00116 second.
" 12, Ans. 155.6 volts; 4.4 kilowatts.

On pages 197–198:

Problem 13, Ans. 65.1°.
" 14, Ans. 89.7°.
" 15, Ans. 174.4°.

On pages 202–204:

Problem 16, Ans. 0.840; 32.9°.
" 17, Ans. 110 volts; 100 amperes; 11 kilowatts; 22 kilowatts.
" 18, Ans. 220 volts; 150 amperes; 16.5 kilowatts.
" 19, Ans. 1,628 volts; A 29.77 kilowatts; B 146.4 kilowatts.
" 20, Ans. A 30.89 kilowatts negative; B 122.8 kilowatts positive.
" 22, Ans. 250.0 amperes.

On pages 208–209:

Problem 24, Ans. (*a*) 45.24 ohms; (*b*) 132.7 ohms; (*c*) 102.7 cycles per second.
" 25, Ans. 0.0270 henry.
" 26, Ans. 29.67 microfarads; 1.6 ohms.
" 27, Ans. R = 10.34 ohms; X = 3.75 ohms; impedance = 11 ohms.
" 28, Ans. R = 5.13 ohms; X = 5.13 ohms; impedance = 7.33 ohms.

On pages 213–214:

Problem 29, Ans. (*a*) 112.6 cycles per second; (*b*) 6224 volts.
" 30, Ans. x = 40 microfarads; 6.634 ohms; 0.6634 ohm.
" 31, Ans. x = 40 microfarads; 1.33 ohms.

On pages 218–219:

Problem 35, Ans. 0.80.

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